ENGR 4220/5220: Control Systems

Professor Hill

University of Detroit Mercy, Winter 2014

Homework #5
Assigned: January 30, 2014
Due: February 6, 2014

Read Sections 5-1 to 5-3 of the book.
Recommended example problems: A-5-1, A-5-2, A-5-3, A-5-4

1. (20 points) Problem B-5-4, page 240. Note that since there are two inputs your **B** matrix will have two columns and since there are two outputs your **C** matrix will have two rows.

2. (10 points) Problem B-5-6, page 242.

3. (20 points) Consider the system given by the following state equations:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & -2 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u$$

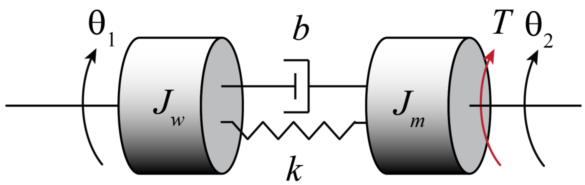
   (a) Find the transfer function for an input of u and an output of x_1.

   (b) Find the transfer function for an input of u and an output of x_2.

   Note: you will have to define your output equation in each case before you can find the transfer function.

4. (20 points) Consider the provided figure that models the driveline of an electric vehicle. The driveline connects the vehicle drive motor to the wheels and is modeled as a torsional spring and damper with stiffness k and damping b. In the figure, J_w represents the rotational inertia of a wheel, while J_m represents the lumped inertia of the motor and transmission. $T(t)$ is the torque reflected from the motor through the gear train to the drive shaft, while $\theta_1(t)$ is the angular displacement of the wheel and $\theta_2(t)$ is the angular displacement of the other side of the drive shaft. The road force is neglected using the assumption that the moment it applies to the wheels is small compared to the moment generated by the drive shaft. In electric vehicles, the driveline does not have as much damping as in a conventional vehicle driveline due to the absence of a clutch and torque converter. This leads to problems with driveline oscillation that adversely affects drivability and passenger comfort. It is, therefore, crucial that the driveline be modeled and analyzed.

Determine the equation(s) of motion for the driveline system shown below.



5. (30 points) Combining the equations from the previous problem and rewriting them in terms of the twist of the drive shaft $\delta\theta(t)$ gives the following:

$$J_m \delta\ddot{\theta}(t) = T(t) - \left(1 + \frac{J_m}{J_w}\right)\left(b\delta\dot{\theta}(t) + k\delta\theta(t)\right)$$

(a) Determine the transfer function of the above if $T(t)$ is considered the input and $\delta\theta(t)$ is considered the output. Include possible units for the resulting transfer function.

(b) Another issue with electric vehicles is that an electric drive motor can generate an almost instantaneous torque to the driveline system, whereas the dynamics of an internal combustion engine are much slower. The rapid application of torque can excite high frequency resonances in the driveline. Plot the response $\delta\theta(t)$ for the situation where $T(t)$ is modeled as an impulse. Assume $J_m = 3.0$ kg·m^2/rad, $J_w = 3.2$ kg·m^2/rad, $k = 1300$ N·m/rad, and $b = 10.5$ N·m·s/rad. You do not need to solve for $\delta\theta(t)$ explicitly, you may use the MATLAB commands **impulse** or **ltiview**.

(c) Repeat part (b) for smaller and larger values of k. How does the response of the driveline change with stiffness k? Explain your observations in terms of the poles of the driveline's transfer function you found in part (a).