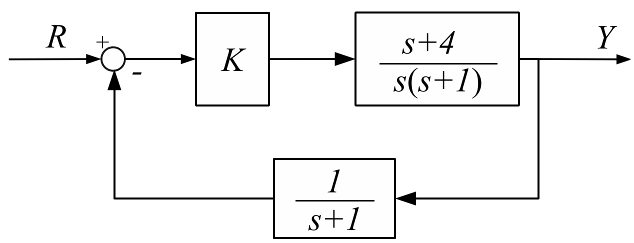
ELEE 4700/5700: Control Systems II

Professor Hill
University of Detroit Mercy, Fall 2012

Homework #4
Assigned: September 27, 2012
Due: October 10, 2012

Read sections 6-1 to 6-8 of the book.
Recommended example problems: A-6-1, A-6-2, A-6-3, A-6-5, A-6-14.

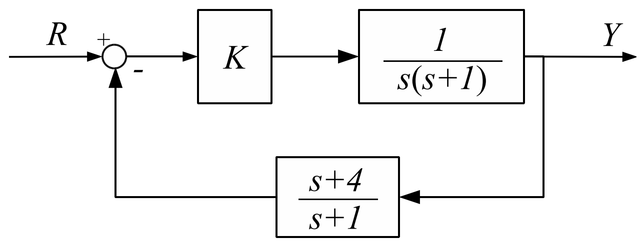
1. (25 points) Plot the root locus for the following closed-loop system by hand. Also analytically determine the range of K values for which the system is stable.



2. (10 points) Consider the system from Problem 1 again.

   (a) Find the closed-loop transfer function for the system. Then calculate and plot the closed-loop poles for the following values of K, $\{0.01, 0.04, 0.1, 0.4, 1, 4, 10, 40\}$. You may find the MATLAB commands **pole** and/or **pzmap** helpful. Does the resulting plot agree with your answer to Problem 1?

   (b) Now consider the feedback system shown below. Find the closed-loop transfer function for this system. How will the root locus for this system compare to the one you found in Problem 1? What does this reveal about the limitations of using the root locus for controller design?



3. (10 points) Again consider the system you drew the root locus for in Problem 1.

   (a) Use the MATLAB command **rlocus** to plot the root locus for this system if it were changed to positive feedback. This can be accomplished by plotting the root locus for the original system multiplied by -1. Attempt to explain why the root locus appearance is different than in Problem 1.

   (b) Use the MATLAB command **rlocus** to plot the root locus for this system if the term $s + 4$ was changed to $4 - s$ (still negative feedback). Attempt to explain why the root locus appearance is different than in Problem 1. Hint: $4 - s = -(s - 4)$.

4. (25 points) Problem B-6-26, page 397. You may find the following steps helpful:

   (a) Write the closed-loop transfer function for this system.

   (b) Rearrange the characteristic equation (denominator = 0) so that it has the form

   $$1 + a\frac{N(s)}{D(s)} = 0$$

   (c) Draw the root locus as a varies from 0 to ∞ by hand in the normal way by treating $N(s)/D(s)$ as the open-loop transfer function.

   (d) Use MATLAB to find the value of a that provides dominant closed-loop poles with a damping ratio of 0.5. The command **rltool** may be most helpful.

5. (25 points) Problem B-6-16, page 395. There are three choices of where to place the zero $-1/T$ associated with this PD controller.

   (a) Show (by plotting the root locus with the MATLAB command **rlocus**) that placing the zero in the right-half plane cannot satisfy the control goals.

   (b) Show (using MATLAB) that placing the zero in between the two open-loop poles cannot satisfy the control goals either.

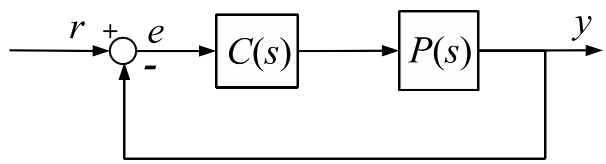
   (c) Show analytically how you chose your values of T and K. Plot the root locus for these values (using MATLAB) and indicate the resulting closed-loop poles.

6. (30 points) Given a plant with the transfer function

   $$P(s) = \frac{s + 4}{s^3 + 6s^2 + 11s + 6}$$

   the aim is to design a controller $C(s)$ that gives minimal steady-state error, a maximum overshoot of 10%, and a 2% settling time of at most 1 second in response to a step input.

   (a) Show that the open-loop poles are located at $s = -1, -2, -3$.



(b) Plot the region of the complex plane that satisfies the transient requirements (overshoot and settle time) assuming two of the closed-loop poles are dominant. Show that a proportional controller $C(s) = K_P$ cannot meet the transient requirements.

(c) Show that if the controller can be designed to achieve a dominant pair of closed-loop poles at $s = -4 \pm 4j$, this would satisfy the transient requirements.

(d) Design $C(s)$ as a lead compensator to place the dominant poles at $s = -4 \pm 4j$.

(e) Plot the step response of the closed-loop system and comment on whether or not all of the requirements are actually met (transient and steady state). Why might the requirements not be met?

7. (30 points) Consider a unity feedback system as in the previous problem, but with the plant

$$P(s) = \frac{K}{(s+2)(s+4)}.$$

It is desired that the system have an overshoot of 4.32% and that its steady-state error be improved. A lag compensator of the following form is to be used.

$$C(s) = \frac{s+0.5}{s+0.1}$$

(a) Find the value of the gain K required to achieve the desired overshoot for the feedback system with the lag compensator and without the lag compensator, assuming two of the closed-loop poles are dominant.

(b) What is the system type of both the compensated and uncompensated systems? Find the steady-state error to a step input for both the compensated and uncompensated systems.

(c) Estimate the resulting settle time for the both the compensated and uncompensated systems.

(d) Plot the step response for the compensated and uncompensated systems. Can you explain what you see?

(e) How could you correct the objection you noticed in Part (d).