ENGR 4220/5220: Control Systems

Professor Hill

University of Detroit Mercy, Winter 2014

Homework #4

Assigned: January 23, 2014

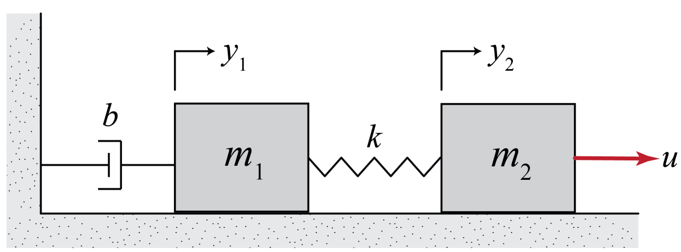
Due: January 30, 2014

Read Chapter 3 and Chapter 4 of the book.

Recommended example problems: A-3-15, A-4-2, A-4-3, A-4-10, A-4-19

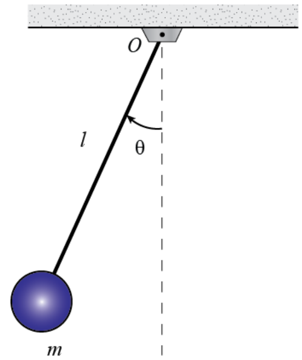
1. (30 points)
   (a) solve Problem B-3-13, page 103.
   (b) Additionally, find the transfer function for the system where θ_i is the input and θ_o is the output.
   (c) Then find the time response $\theta_o(t)$ for a unit step input $\theta_i(t)$ assuming $J = 100$ kg-m^2, $b = 200$ N-m-s, and $k = 500$ N-m.
   (d) Plot the system step response (i.e. $\theta_o(t)$) using the MATLAB command **step** or **ltiview**. This result should match your answer in Part (c).

2. (20 points) Consider the figure shown below.



   (a) Write the equations of motion for the shown system where u is an input force.
   (b) Find the transfer function for an input of $u(t)$ and an output of $y_1(t)$. Note, you will need to combine the two equations of motion you found in Part (a). Include units assuming a metric system is employed.

3. (20 points) Reconsider the following simple pendulum (from Problem 4 of Homework #3) where $l = 2$ m and $m = 4$ kg. The mass of the rod can be neglected.



(a) Restate the fully nonlinear model of the pendulum (assuming $J = ml^2$) as well as the linearized approximation of the model. Based on the linearized model, also restate the expected behavior of the pendulum's free response.

(b) Build a Simulink model for the full nonlinear equation of motion from part (a). Simulate the free response of the system (no forcing input) for initial conditions of $\theta(0) = 5°$ and $\dot{\theta}(0) = 0$ using the fixed step size ode1 (Euler) solver with time steps of 0.1, 0.01, and 0.001 seconds. How do your results compare to the prediction of Part (a)? What explains these results?

If you need help with Simulink, you may refer to the tutorial at
$http://ctms.engin.umich.edu/CTMS/index.php?aux = Basics\_Simulink$.

4. (30 points) Consider a mechanical system defined by the transfer function

$$\frac{X(s)}{U(s)} = \frac{1}{s^2 + 2s + 10}$$

The system is initially at rest, and is subjected to a unit step input $u(t) = 1(t)$.

Answer the following questions *without* solving for $x(t)$ explicitly.[1]

(a) What are the poles of the transfer function?

(b) Find $X(s)$. What is the final (steady-state) value of the output $x_{ss} = \lim_{t \to \infty} x(t)$?

(c) Estimate how long does the output take to reach its final value (within 2%)?

(d) What is the frequency of oscillation?

(e) How many times does the output oscillate before settling down to its final value?

[1] You may wish to check your answer by solving the equation and/or plotting the step response with MATLAB but do not turn these in.