ENGR 4220/5220: Control Systems

Professor Hill

University of Detroit Mercy, Winter 2014

Homework #12

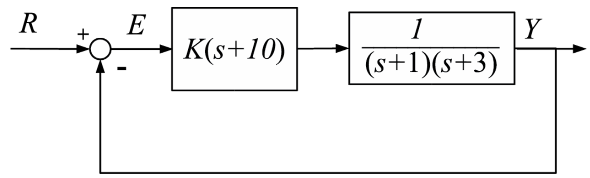
Assigned: April 3, 2014

Due: April 10, 2014

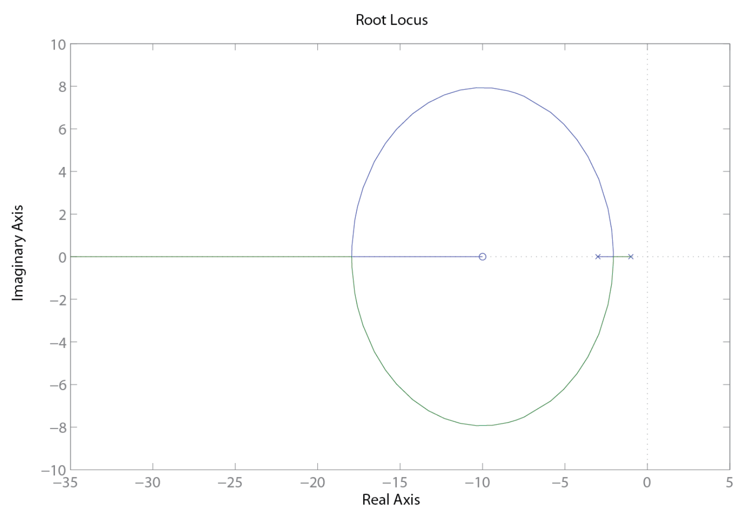
Read Sections 10-8, 10-9, and 11-1 to 11-3 of the book.

Recommended example problems: A-10-11, A-10-12, A-10-13, A-10-14, A-11-1, and A-11-4

1. (15 points) Consider the unity feedback system shown below.



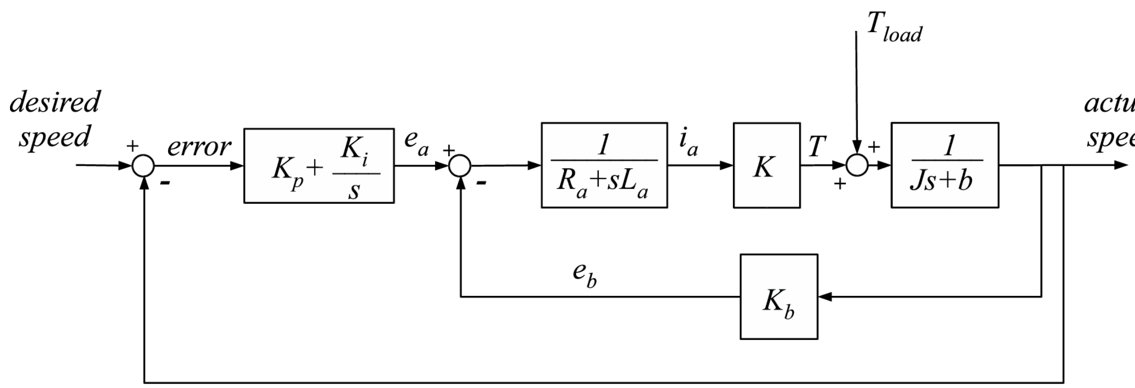
The root locus for the above system is given below as the gain K is varied from 0 to ∞.



(a) Examining the root locus plot shown above, approximately where would you place the system's closed-loop poles in order to achieve the minimum possible peak time t_p for this system's closed-loop step response? Justify your answer.

(b) Let's say that you choose K to place the closed-loop poles at the locations identified in Part (a) and find that the resulting overshoot is larger than is acceptable. How do you think you should modify the gain K to reduce the resulting overshoot? Justify your answer.

2. (35 point) Consider the following block diagram of a speed control system for a motor. It is desired to design the controller to achieve a peak time less than 0.15 seconds and a maximum percent overshoot less than 15% for the closed-loop step response.



Reducing the inner-loop of the above block diagram, the DC motor plant can be represented by the following transfer function:

$$\frac{\Omega(s)}{E_a(s)} = \frac{K}{(Js+b)(sL_a+R_a)+KK_b} = \frac{41,100}{s^2+200s+3100}\frac{\text{rad/s}}{\text{V}} \tag{1}$$

(a) Explain why an algebraic approach to pole placement may be difficult to employ in this case.

(b) Sketch the region in the complex plane where the closed-loop poles must be located to satisfy the given requirements.

(c) Using a root-locus argument, can the <u>dominant</u> closed-loop poles be placed in the desired region employing a proportional controller? Justify your answer. You may use MATLAB.

(d) Recall that a PI controller includes a zero and a pole at the origin ($K_p + K_i/s = K(s+z)/s$). Use the MATLAB command **rlocus** to find a zero location that will place the closed-loop poles in the desired region. Provide a value for z and K that you found to work (the command **rlocfind** may be helpful). What are the corresponding values of K_i and K_p?

(e) Based on your resulting closed-loop pole locations, calculate the expected settle time, peak time, and percent overshoot if the dominant poles belonged to a canonical second order system.

(f) Plot the closed-loop step response for the given plant and compensator you found in Part (d). How does the response compare to your prediction from Part (e)? Explain any differences.

3. (20 points) For the transfer function $G(s) = \frac{10}{s(0.1s+1)}$ evaluate its magnitude and phase at $\omega = 2, 5, 10, 20, 50, 100$ and 200 rad/sec. Plot the magnitude in dB and the phase in degrees versus $\log \omega$.

4. (30 points) Sketch the straight-line approximations of the Bode diagrams of the following two transfer functions by hand. Check your results with the `MATLAB` command **bode**.

$$G_a(s) = \frac{5000}{(s+70)(s+500)} \qquad G_b(s) = \frac{3000(s+100)}{s(s+10)(s+40)}$$