ENGR 4220/5220: Control Systems

Professor Hill

University of Detroit Mercy, Winter 2014

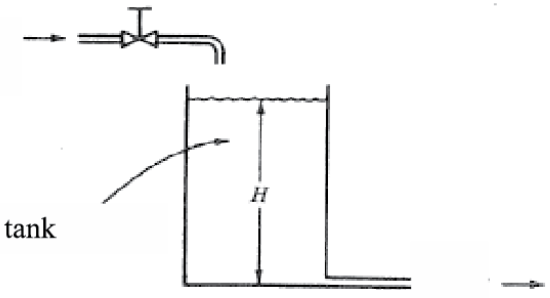
Homework #2
Assigned: January 9, 2014
Due: January 16, 2014

Read Sections 2.1 and 2.3 of the book.
Recommended example problems: A-2-2, A-2-3, A-2-5, A-2-14

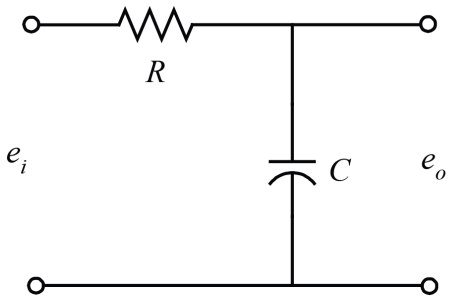
1. (10 points) Turn in a plot of $y(t) = 5 - 2e^{-2t}\sin(6t)$ for $t \in [0, 4]$ with an appropriate title, axis labels, and your name inserted in the plot somewhere (use `MATLAB` to put your name in the plot, do not write it by hand). Be sure to use enough points in the t vector so that the plot is smooth. You may refer to the web page `http://ctms.engin.umich.edu` if you need extra help. Specifically, you can go to the `BASICS > MATLAB` link, or the `INDEX > Extras > Plot` link.

2. (20 points)

   (a) In class we examined an automotive cruise control system as a block diagram consisting of a "Controller," a "Plant," a "Sensor," and an "Actuator." Generate an analogous diagram for a closed-loop control system to maintain the water level in a tank that has a single inlet valve (that can be controlled) and an unknown leak. An example of a closed-loop system such as this is the tank on a toilet. Identify all of the blocks of the closed-loop system and the various signals in the loop such as the "Reference," the "Output," and the "Disturbances." In a sentence or two, explain the logic the controller might use.



   (b) Explain the advantages and disadvantages of the control system described above compared to an open-loop version of the system. Be specific to this example.



3. (25 points) Consider the above RC circuit which is one implementation of a low-pass filter.

   The differential equation describing this circuit is given below, where $e_i(t)$ is the input voltage applied to the circuit and $e_o(t)$ is the output voltage across the capacitor C.

$$RC\dot{e}_o(t) + e_o(t) = e_i(t)$$

   (a) Consider that the capacitor has built up a charge so that the voltage across it is 3 Volts when at time $t = 0$ the voltage applied to the circuit is removed ($e_o(0) = 3$ and $e_i(t) = 0$). If $R = 5$ Ohms and $C = 2$ Farads, determine how the output voltage across the capacitor $e_o(t)$ changes as a function of time. In other words, solve the given differential equation for $e_o(t)$.

   (b) How would you change the resistance and/or capacitance of the circuit to increase the rate at which the output voltage decays? Explain.

4. (15 points) Use the tables on pages 18-20 to find the Laplace transforms of the following functions:

   (a) $f_a(t) = t + e^{-4t} \sin 2t$

   (b) $f_b(t) = \int (1 - e^{-5t})\, dt$, assume zero initial conditions

   (c) $f_c(t) = t^2 e^{-5t}$

5. (10 points) Given the following function $F(s)$, apply the Initial Value Theorem to find $f(0+)$.

$$F(s) = \frac{5(s+3)}{s(s+2)(s+5)}$$

6. (20 points) Consider the following Laplace function

$$F(s) = \frac{s+3}{s^2+6s+13}$$

   (a) Find the poles of the above function $F(s)$. What does this tell us about the corresponding time function $f(t)$?

   (b) Obtain $f(\infty)$ using the final value theorem. Does this agree with the answer you found in Part (a)?

The following problem is for students enrolled in ENGR 5220 only:

7. (20 points) Find the solution $y(t)$ of the differential equation $10\dot{y} + 5y = 2t$ if $y(0) = 3$.