ENGR 4220/5220: Control Systems

Professor Hill

University of Detroit Mercy, Winter 2014

Homework #3
Assigned: January 16, 2014
Due: January 23, 2014

Read Sections 2.4, 2.5 and 3.1 to 3.3 of the book.
Recommended example problems: A-2-16, A-2-17, A-2-19, A-2-21, A-3-8, A-3-13, A-3-16

1. (20 points)

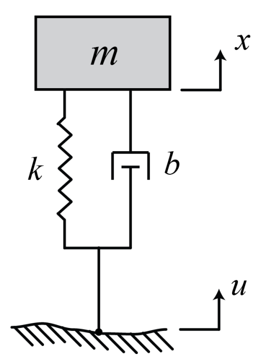
   (a) Apply the inverse Laplace transform to the following to find $x(t)$:

   $$X(s) = \frac{s+5}{(s+1)(s+3)}$$

   (b) Apply the inverse Laplace transform to the following to find $y(t)$:

   $$Y(s) = \frac{2s+1}{s(s^2+2s+3)}$$

2. (25 points) Consider the simplified quarter-car model of an automobile suspension given below where m is one quarter of the total mass of the vehicle, b is the damping in the suspension, and k is the stiffness of the suspension.

   The differential equation describing this system is

   $$m\ddot{x} + b\dot{x} + kx = u$$

   where $x(t)$ is the displacement of the car body and $u(t)$ is the displacement of the tire, that is, the height of the road surface.

Consider the response of our suspension subject to a sharp bump in the road, $u(t) = \delta(t)$. It turns out that the impulse response of a system is equivalent to the free response of a system to an initial condition on its velocity. In other words, our situation is described by the following differential equation where for simplicity $m = 1$, $b = 2$ and $k = 4$.

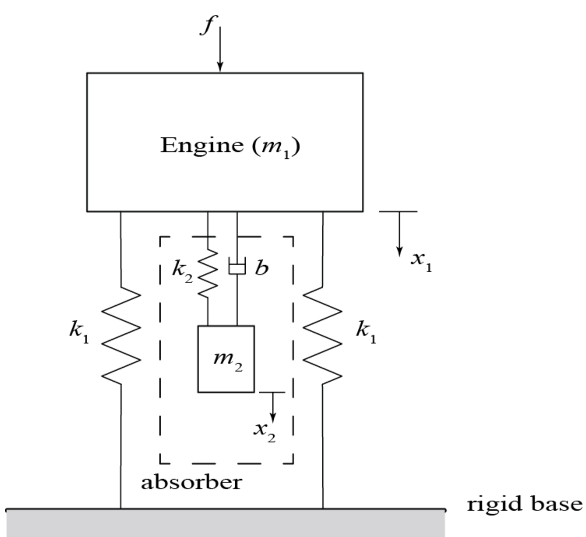
$$\ddot{x} + 2\dot{x} + 4x = 0, \text{ where } x(0) = 0, \dot{x}(0) = 1$$

(a) From the above differential equation, find $X(s)$. Determine the poles of $X(s)$. What does this tell us about the response of our car body $x(t)$? Can you apply the final value theorem to find $x(t)$ as $t \to \infty$? Explain.

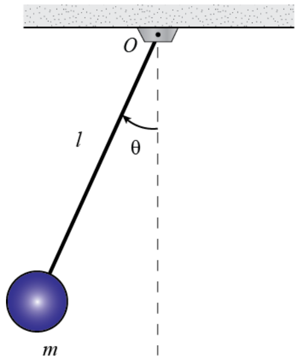
(b) How would you change the system parameters (m, b, and k) to make any oscillations in the car body damp out more quickly? Justify your answer.

(c) Finish solving the differential equation, that is, find $x(t)$. Does the function $x(t)$ you found agree with your answer to Part (a)? Explain.

3. (15 points) Consider the internal combustion engine m_1 shown below that is attached to the automobile frame via elastic mounts that are modeled here as springs with stiffness k_1. The vibration induced by the reciprocating parts of the engine is modeled as the external force f and the automobile frame can be considered as fixed. One approach to reducing the vibration induced by the engine on the frame is to connect a second smaller mass m_2 to the primary mass via an elastic member modeled here as having stiffness k_2 and damping b. Write the differential equation(s) that govern the behavior of this system as described.



4. (20 points) Consider the following simple pendulum where $l = 2$ m and $m = 4$ kg. The mass of the rod can be neglected.



   (a) Find the equation of motion for this pendulum system. The moment of inertia of the pendulum about its point of attachment is $J = ml^2$.

   (b) Approximate the nonlinear differential equation from part (a) assuming the angle θ is small. What would you expect the free response of this linearized system to look like?

   (c) If you wished to reduce the frequency with which the pendulum oscillates, how would you modify the pendulum's design? Explain.

5. (20 points) A control pedal for an airplane is modeled as shown. The lever may be modeled as a massless shaft and the pedal as a lumped mass $m = 0.2$ kg at the end of the shaft where $l = 13$ cm. A spring of stiffness $k = 50$ N/m and a damper with damping $b = 20$ Ns/m are attached as shown where $l_1 = 6$ cm and $l_2 = 7$ cm. The spring is unstretched when the lever is in the vertical position ($\theta = 0$). Determine the equation of motion for the pendulum assuming the angular motion is small such that we can approximate the spring and damper as moving horizontally.