ENGR 4220/5220: Control Systems

Professor Hill

University of Detroit Mercy, Winter 2014

Homework #10

Assigned: March 20, 2014

Due: March 27, 2014

Read Sections 10-3, 10-4, and 10-6 of the book.

Recommended example problems: A-10-4, A-10-5, A-10-6, and A-10-8

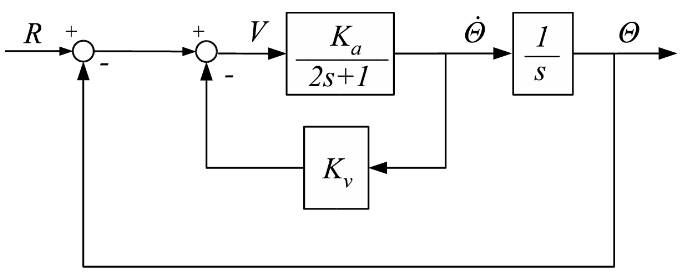
1. (25 points) Consider the system shown below which involves velocity feedback. Determine the values of the amplifier gain K_a and the velocity feedback gain K_v so that the following specifications are satisfied:

   (a) Damping ratio of the closed-loop poles equals 0.5

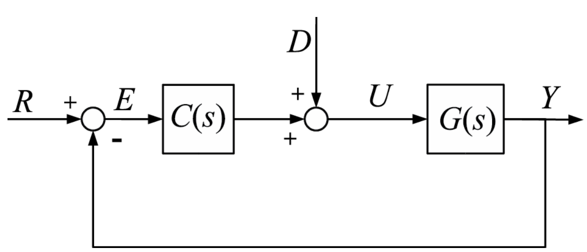
   (b) Settling time is less than 2 seconds

   (c) Steady state error to a unit ramp input is less than 0.02

   (d) $0 < K_v < 1$



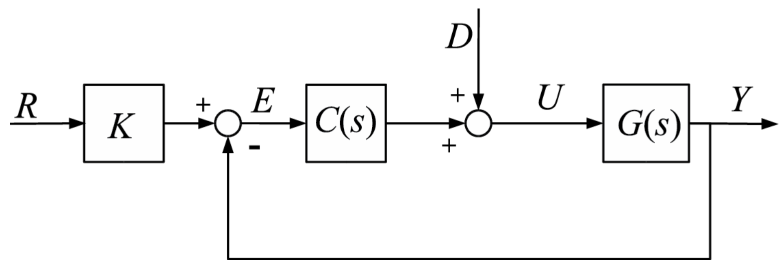
2. (30 points) A closed-loop control system is shown in the figure below.



   (a) If the controller is a simple proportional controller $C(s) = 5$ and the plant is $G(s) = \frac{1}{s+2}$, what is this system's type? Can the system track a unit step reference input R with zero steady-state error? If no, determine the error.

(b) If integral control is added to the controller such that $C(s) = 5 + \frac{1}{s}$, what is this system's type? Can the system track a unit step reference input or a unit ramp reference input R with zero steady-state error? If no, determine the error.

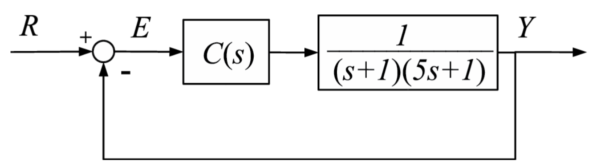
(c) Rather than adding integral action to the controller, let the controller remain $C(s) = 5$ and instead add a constant pre-gain K as shown in the figure below. Determine the value of K that will allow the system to track a unit step reference with zero-steady state error. Can the system track a unit ramp reference input with zero steady-state error? If no, determine the error.



(d) Simulate the systems described in part (b) and part (c) above for both a unit step reference and a unit ramp reference.

(e) Discuss the tradeoffs between employing integral control and a constant pre-gain to reducing steady-state error.

3. (30 points) Consider the following unity-feedback system.



We will consider this system under P, PD, and PID control for the parameters $K_P = 19$, $K_I = 0.5$, and $K_D = 2$.

(a) For each of the above controllers, determine the closed-loop transfer function from the reference input R to the output Y. You may use the MATLAB command **feedback** to calculate the closed-loop transfer functions.

(b) Determine the poles, zeros, and DC gain for each of the closed-loop transfer functions of Part (a). You may use the MATLAB commands **pole** and **zero**.

(c) Use MATLAB to generate a step response for each of the closed-loop systems. Turn in a plot of the output responses versus time and qualitatively compare the results. What was the effect of adding the derivative term? What was the effect of adding the integral term? Does this agree with your Part (b)? Explain.

4. (15 points) Consider the plant given below:

$$G(s) = \frac{1}{s(s+1)(s^2+4s+5)}$$

(a) Find the open loop poles of $G(s)$.

(b) Find the closed-loop transfer function for $G(s)$ under the proportional control K_p.

(c) Calculate and plot the closed-loop poles for the closed loop system as K_p varies from 0 to 10. You may use the MATLAB commands **pzmap** and **hold**.

(d) Based on Part (c) estimate the value of K_p for which the closed-loop system becomes unstable. Also, estimate the frequency of oscillation of the system for this value of K_p.