Sample Midterm
ENGR 4220/5220, Control Systems
Professor Hill
University of Detroit Mercy, Winter 2014

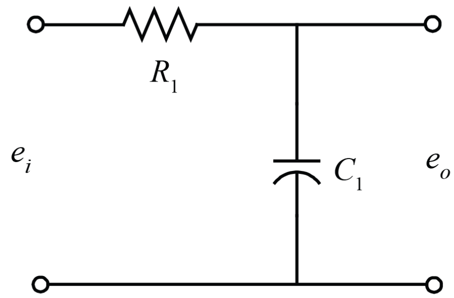
Name: ______________________________

Date: ______________________________

Scores:	Problem 1	________	30 points
	Problem 2	________	25 points
	Problem 3	________	20 points
	Problem 4	________	25 points
	total	________	100 points

The exam is closed book and closed notes. One sheet of notes (8.5×11), one side, and a calculator are allowed. An abbreviated Laplace transform sheet consisting of some pairs and properties will be provided on the exam. Solutions to this sample exam will be posted on the course website.

1. (30 points) Consider the following RC-circuit implementation of a low-pass filter.

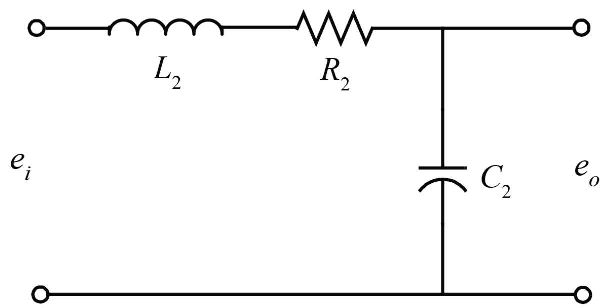


(a) The transfer function for this circuit is given below. Employing this model, find the output response of the circuit $e_o(t)$ for a step input of magnitude E to the circuit, $e_i(t) = E \cdot 1(t)$.

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{R_1 C_1 s + 1}$$

(b) If $R_1 = 5\ \Omega$ and $C_1 = 2$ F, determine the settling time t_s for the filter, that is, the time it takes for the circuit to reach and stay withing $\pm 2\%$ of its final value.

(c) An alternative implementation of a low-pass filter is given below. Find the transfer function for this circuit for an input of $e_i(t)$ and an output of $e_o(t)$.



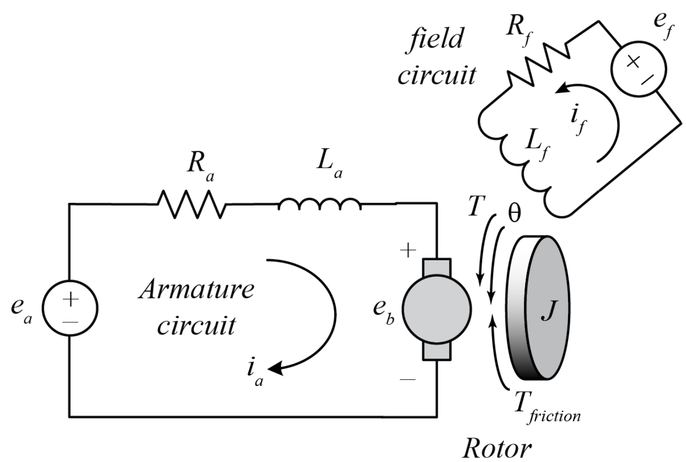
(d) You wish to design the filter in Part (c) to achieve similar performance to the filter in Part (a). Specifically, if $L_2 = 5$ H, then find the resistance R_2 and capacitance C_2 so that:

i. The two filters have the same settling time t_s (corresponds to the filter's pass band).

ii. The two filters reach the same steady-state value for a constant input.

iii. The second filter has a maximum percent overshoot equal to 5%.

2. (25 points) Consider the *field-controlled* DC motor shown below. In this motor the armature voltage e_a is constant and the speed of the motor is controlled by varying the strength of the magnetic field which is controlled by the field voltage e_f. There is, however, still a back emf in the armature circuit $e_b = K_b\dot{\theta}$ due to the rotation of the armature. Since the field circuit is stationary it does not move in the magnetic field and thus there is no back emf in the field circuit. The expression for motor torque is similar to the armature controlled case, but the armature current cannot be lumped into the constant because it changes due to the back emf, therefore, $T = Ki_f i_a$. The motor also includes frictional torque modeled as $T_f = b\dot{\theta}$.



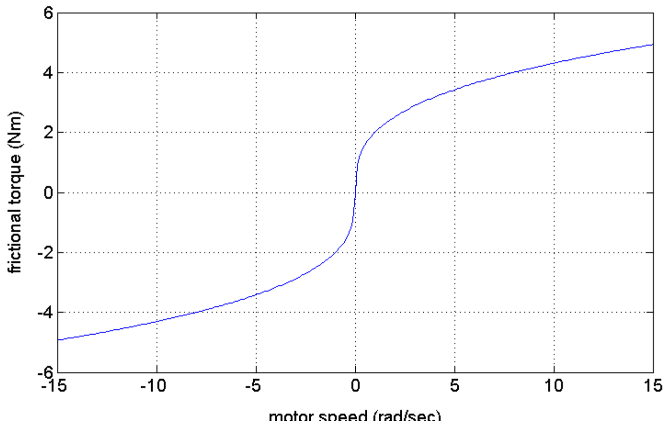
(a) Write the differential equations of the DC motor for the field controlled case (you should have three equations).

(b) The equations from Part (a) are considered nonlinear because the motor torque is a function of both the armature current and the field current, $T = Ki_f i_a$. Therefore, these equations are very difficult to find a closed-form solution to. With this in mind, how might you analyze this system?

(c) Now assume that the back emf is small enough that the armature current can be considered constant. Therefore, $T = \hat{K} i_f$. Find the transfer function $G(s)$ for an input of field circuit voltage $e_f(t)$ and an output of motor velocity $\omega(t)$ where $\omega(t) = \dot{\theta}(t)$. Hint: you will only need to use two of your three equations from Part (a). Also remember units on your transfer function.

(d) Determine the poles of the transfer function $G(s)$. What do the poles tell you about the effect of the motor's components on the motor's dynamic response?

3. (20 points) Consider the following empirical data showing the frictional torque in a motor as a function of the motor's angular speed, $T_{fric} = f(\omega)$. It is possible to fit a nonlinear function to the friction data such that $f(\omega) = 2\sqrt[3]{\omega}$. If you are able to solve the problem using the empirically-derived graph directly (no function), you will receive 5 points extra credit. The resulting equation of motion for the mechanical portion of this motor is then $T - f(\omega) = J\dot{\omega}$ where T is the torque generated by the motor and J is its rotational inertia.

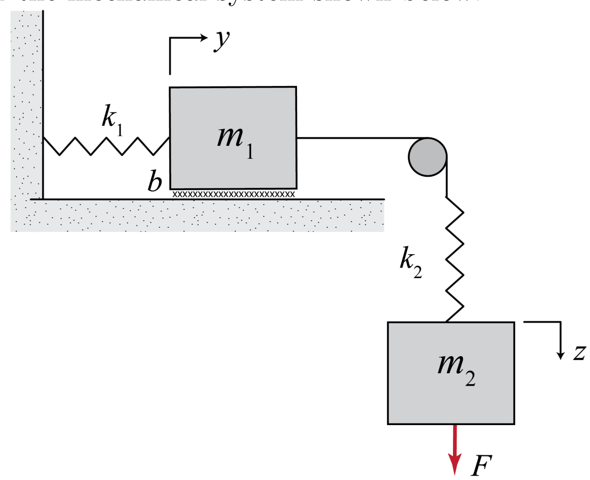


(a) If the motor is to operate primarily with a constant speed of 10 rad/sec, identify a good equilibrium point $(\bar{T}, \bar{\omega})$ to linearize the system about.

(b) Linearize the given nonlinear differential equation about the equilibrium point you found in Part (a). Express the linearized equation using numerical values assuming $J = 2$.

(c) Discuss the advantages and disadvantages of employing the linearized model you just found as compared to a differential equation with a nonlinear friction model.

4. (25 points) Consider the mechanical system shown below.



(a) Draw a free-body diagram for m_1 and m_2.

(b) Write the equations of motion. State explicitly the reference positions for y and z.

(c) What is the minimum number of state variables needed for this system? Explain. Choose a set of state variables.

(d) The input to the system is F and the output is y. Put the equations from Part (b) into matrix state-space form.