ENGR 4220/5220: Control Systems

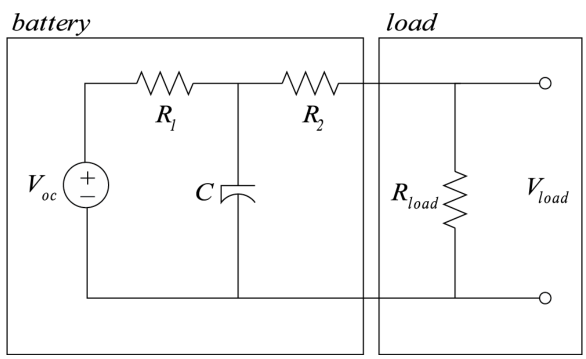
Professor Hill

University of Detroit Mercy, Winter 2014

Homework #6
Assigned: February 6, 2014
Due: February 13, 2014

6-1 to 6-3 of the book.
Recommended example problems: A-6-2, A-6-5, A-6-6, A-6-7, A-6-8, A-6-10, A-6-11, and A-6-12

1. (25 points) Consider the following circuit equivalent model of a Li-ion battery used in an electric vehicle. V_{oc} is called the open-circuit voltage and can be thought of as the voltage source for the battery. The resistors and capacitors are included to approximate the dynamics of the battery when a load is applied to it. For the purposes of this problem, the load on the battery is modeled as a simple resistance.

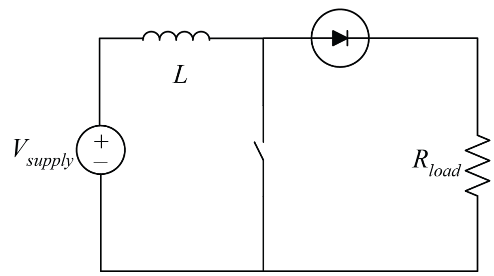


   (a) Obtain a mathematical model for this circuit.

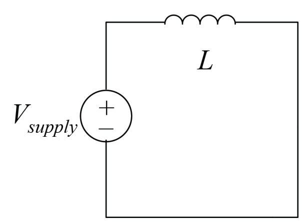
   (b) If you have not done so already, transform the model from Part (a) into differential equations written in terms of charge q, where current is the time derivative of charge $i = \dot{q}$.

   (c) Find the transfer function model for this circuit for an input of V_{oc} and an output of V_{load}, where V_{load} can be expressed in terms of R_{load} using Ohm's law.

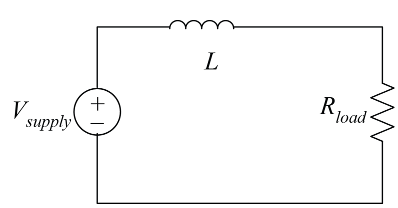
2. (35 points) Below is given a simple model of a "boost" power converter that takes a DC voltage as an input and outputs a higher DC voltage. One application of a boost DC-DC converter is to reduce the number of battery cells that must be placed in series in order to achieve a desired output voltage. For example, the 2nd generation Toyota Prius uses a 500 V motor that would require 417 battery cells for power if no converter was used. Rather, the Prius uses only 168 cells and boosts the output voltage from 202 V to 500 V using a DC-DC converter.



(a) When the switch in the above circuit is closed the converter is in its "ON" state and the current from the supply loops as shown below thereby adding energy to the inductor. For the converter as drawn, no current will flow through the load in this state. Model the converter in the state shown below as a differential equation. Then determine the current through the inductor as a function of time assuming that the initial current is 0 and that the supply voltage is a constant E Volts DC.

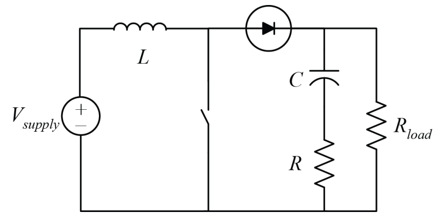


(b) When the switch in the circuit is opened and the converter is in its "OFF" state the diode is forward biased and the circuit behaves like the following. Model the converter in this state again as a differential equation. Next determine the current through the inductor again as a function of time assuming the supply voltage is a constant E volts DC. This time do not assume that the initial current is 0.

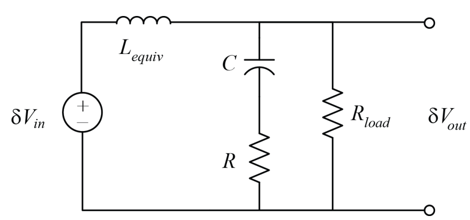


(c) Draw a graph of what the inductor current might look like versus time if the switch is opened and closed with a constant frequency. Also draw a graph of the current supplied to the load. In actual implementation, a capacitor would be added to smooth the ripple.

3. (20 points) In Problem 2 you considered a simplified model of a boost power converter connected to a simple resistive load. Below is given a more complex model of a boost converter including a capacitor on the output to filter the ripple associated with the switching of the converter.



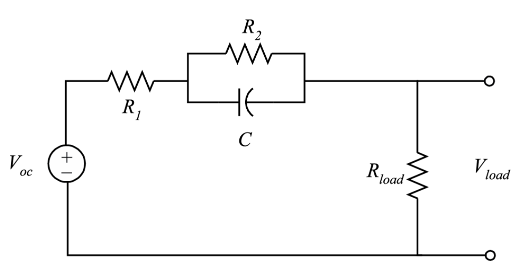
If the switching frequency of the converter is high enough, an average linear model of the converter is often used. One such average model is shown below where the equivalent inductance L_{equiv} is related to the actual inductance L by the relation $L_{equiv} = L/(1-D)^2$ where D is the duty cycle of the switching and δV_{in} and δV_{out} are incremental changes in voltage from some nominal input and output, respectively.



(a) Obtain the transfer function $\Delta V_{out}(s)/\Delta V_{in}(s)$ for this circuit using the complex-impedance method. You do not need to substitute for L_{equiv} in your final answer.

(b) The model employed in Part (a) employed some simplifying assumptions. Discuss the tradeoffs associated with employing this simplified model for your analysis.

4. (20 points) Consider the following circuit equivalent model of a Li-ion battery used in an electric vehicle. V_{oc} is called the open-circuit voltage and can be thought of as the voltage source for the battery. The resistors and capacitors are included to approximate the dynamics of the battery when a load is applied to it. For the purposes of this problem, the load on the battery is modeled as a simple resistance.



Obtain the transfer function for this circuit using the complex-impedance method where V_{oc} is considered the input and the current through the load I is considered the output.