ELEE 4700/5700: Control Systems II

Professor Hill

University of Detroit Mercy, Fall 2012

Homework #2

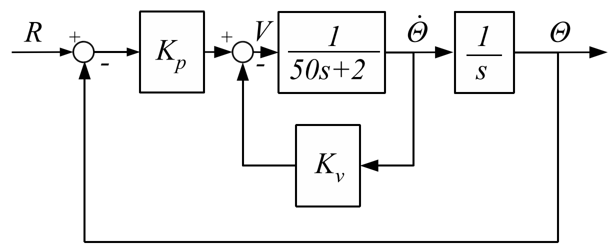
Assigned: September 13, 2012

Due: September 20, 2012

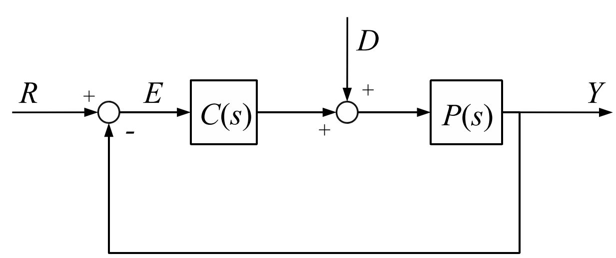
Read sections 2.1 to 2.3, and 5.1 to 5.4

Recommended example problems: A-2-1, A-2-2, A-5-3, A-5-4, A-5-8

1. (10 points) Consider the motor position control system shown below that employs velocity and position feedback. Find the transfer function for an input of R and an output of Θ.



2. (20 points) Consider the following unity-feedback system with disturbance.



   (a) Suppose the given system has a plant and controller given by:

$$P(s) = \frac{1}{s-1}, \qquad C(s) = \frac{s-1}{s+1}$$

In this instance the controller was designed to cancel an unstable pole in the plant. Find the "gang of four" transfer functions for this system and the total output $Y(s)$ for simultaneous inputs of $R(s)$ and $D(s)$.

(b) Explain why the controller designed in Part (a) would be a bad choice.

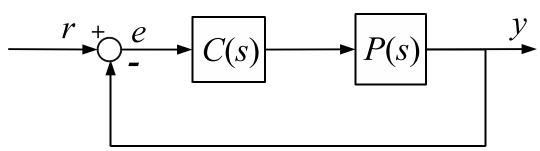
(c) Now suppose the given system has a plant and controller given by:

$$P(s) = \frac{s-1}{s+1}, \qquad C(s) = \frac{1}{s-1}$$

In this instance the controller was designed to cancel a non-minimum phase zero in the plant. Again find the "gang of four" transfer functions for this system.

(d) Explain why the controller designed in Part (c) would be a bad choice.

3. (25 points) Consider the unity-feedback system shown below. In this problem we wish to demonstrate the **Internal Model Principle** which states that a unity-feedback system can robustly track a reference input if the forward path transfer function $L(s) = C(s)P(s)$ includes a "copy" of the reference signal. You may have encountered a special case of this principle when studying the notion of **system type**.



(a) Consider the above system for the following proportional controller and first-order plant:

$$P(s) = \frac{1}{s+1}, \qquad C(s) = 5$$

Determine an expression for $E(s)$ when the reference is $r(t) = \cos 3t$. Can you apply the final value theorem to find the steady-state error? Explain. Determine an expression for the steady-state error.

(b) Now consider the same system, but with an alternate controller:

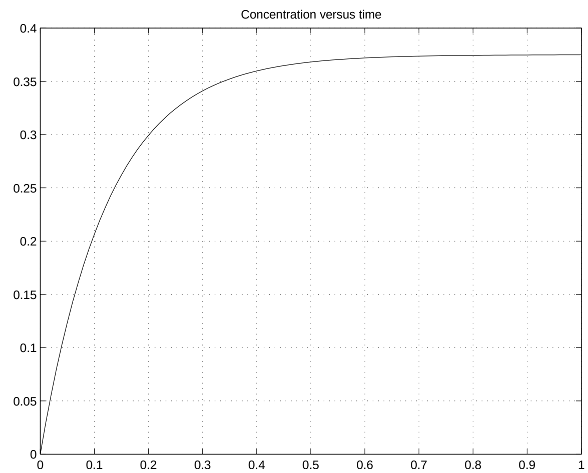
$$P(s) = \frac{1}{s+1}, \qquad C(s) = \frac{s}{s^2+3^2}$$

Determine an expression for $E(s)$ when the reference is $r(t) = \cos 3t$. Can you apply the final value theorem to find the steady-state error? Explain. Determine an expression for the steady-state error.

4. (15 points) A simple model of a chemical process can be described by a first order ordinary differential equation:

$$\dot{x}(t) + a \cdot x(t) = u(t),$$

where x is the chemical concentration and u is the input to the process. The concentration is initially zero (inert mixture). At time $t = 0$ a step input $u(t) = b1(t)$ is applied to the mixture causing a step response of the output concentration x shown in the figure below.



(a) Find the transfer function of the chemical process from the input u to output x in terms of a and b.

(b) Find the numerical values of a and b.

5. (30 points) Consider that a battery in an electric vehicle is installed in a rectangular package such that all but one side of the package is well insulated. It is given that the battery package generates heat at a rate $\dot{q}$ and has some thermal capacity C. Also, the process of the heat escaping through the uninsulated wall is described by a thermal resistance R that is a function of the wall material and its geometry. Therefore, the equation describing the battery temperature T_B is as follows, where T_O is the ambient temperature outside of the package.

$$C\dot{T}_B = \frac{1}{R}(T_O - T_B) + \dot{q}$$

Rewriting the given differential equation in terms of the difference between the battery's temperature and the ambient temperature, $d = T_B - T_O$, the above equation becomes

$$C\dot{d} + \frac{1}{R}d = \dot{q}$$

assuming that the ambient temperature T_O is constant. Assuming values for C and R the above differential equation becomes

$$\dot{d} + \frac{1}{2}d = \dot{q}$$

(a) If we assume that the temperature differential d is initially 1° C and the battery with cooling system is being operated such that it is generating (and dissipating) heat in a manner that can be approximated as $\dot{q}(t) = \sin(15t)$ W, find an expression for temperature differential in the Laplace domain $D(s)$. Based on your expression $D(s)$, how do you expect the temperature differential to behave in time? Note, you do not need to determine $d(t)$.

(b) Now simulate the above situation assuming a constant ambient temperature of $T_O = 40^\circ$ C. You may find the following Simulink model helpful. Recall that initial conditions can be set in the 'integrator' block and that the simulation solver parameters can be set from the drop-down menu **Simulation** $>$ **Configuration Parameters** within Simulink. You may find the default simulation time step too large. Turn in a plot of the resulting response.

(c) Take the inverse Laplace transform of the expression for $D(s)$ you found in part (a). The resulting response $d(t)$ should agree with the results of your simulation.