Laboratory Activity #1:
DC Motor Modeling and Identification

ENGR 4220/5220: Control Systems
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University of Detroit Mercy, Summer 2013

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1 Introduction

In this course we have learned that there are two ways to generate a mathematical model for a physical system. The first approach is to derive a mathematical model from first principles using physical laws like Newton's Second Law or Kirchoff's Voltage Law. The second approach is a black-box approach where a mathematical model is found that attempts to match the output observed from the actual physical system for a known input or series of inputs. In this lab we will explore both of these techniques for an armature-controlled DC motor like we have have studied in class. A representation of such a DC motor is shown in Figure 1:

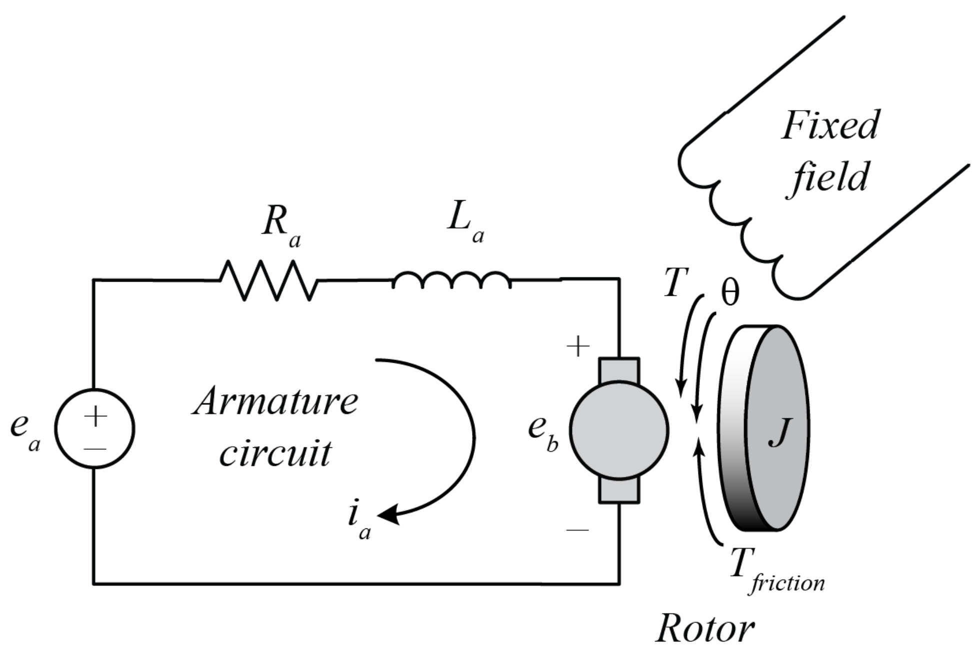

Figure 1: Simplified representation of an armature-controlled DC motor

2 Part A: Model Derivation from First Principles

Assuming a lumped-parameter model like the one shown in Figure 1, recall the following differential equation model of the motor that was derived in class from first principles.

$$
\begin{aligned}
e_a(t) &= R_a i_a(t) + L_a \frac{di_a(t)}{dt} + e_b(t) &(1)\\
T(t) &= J\ddot{\theta}_m(t) + T_{friction}(t) &(2)
\end{aligned}
$$

In the above, the motor torque is proportional to the armature current $T(t) = Ki_a(t)$ and the back emf generated in the armature is proportional to the speed of rotation $e_b(t) = K_b\dot{\theta}_m(t)$. The frictional torque is a little different than what we have modeled in class in that it has a viscous component and a stiction component, $T_{friction} = b\dot{\theta}_m(t) + T_{stiction}\text{sgn}(\dot{\theta}_m(t))$. The stiction component has constant magnitude and opposes the direction of motion as captured by the signum function, sgn. The signum function captures the sign of its argument as demonstrated below:

$$
\text{sgn}(a) = \begin{cases} 1 & \text{if } a > 0 \\ 0 & \text{if } a = 0 \\ -1 & \text{if } a < 0 \end{cases}
$$

From the above equations with $T_{stiction}$ neglected, also recall we formulated an alternative transfer function model of the motor. Assuming an input of armature voltage e_a and an output of motor angular speed $\omega = \dot{\theta}_m$, the following was derived. Note the units.

$$
\frac{\Omega(s)}{E_a(s)} = \frac{K}{(Js+b)(sL_a+R_a)+KK_b}\frac{\text{rad/s}}{\text{V}} \tag{3}
$$

The DC motor that we will be modeling has been shipped with the following parameters identified by the manufacturer:

motor torque constant	K	0.074 Nm/Amp
back emf constant	K_b	0.074 V/rad/s
armature resistance	R_a	0.60 Ω

We will estimate the remaining parameters using the above equations and performing a series of experiments. Note, the parameters provided by the manufacturer could be estimated empirically in a similar manner.

2.1 Experimental Set-up

We begin our experiment by setting up the system that we will use for commanding and observing the drive DC motor and the motor for used generating a load which we will not use in this experiment. Figure 2 illustrates the experimental set-up. The logic that will be used for interacting with the motor and its sensors will be implemented in Simulink. The Simulink model will then be compiled into C code that will run on a dSPACE DSP Controller Board that will run the tasks designed in Simulink in real time. This controller board sends and receives signals from an I/O board that handles the signal-level interface with the hardware. For example, the I/O board performs all of the digital-to-analog conversion and analog-to-digital conversion, the counting of encoder pulses, and so forth. The signals discussed so far carry information, but do not have enough power to drive the motors. The Power Electronics Drive Board connects the motors to a 42-V DC power source and generates pulse-width-modulated (PWM) commands based on instructions from the controller board. Furthermore, the power electronics board protects the system from damage from the high voltage power source and back emf from the electric motors. The power electronics board is powered by a 12-V supply.

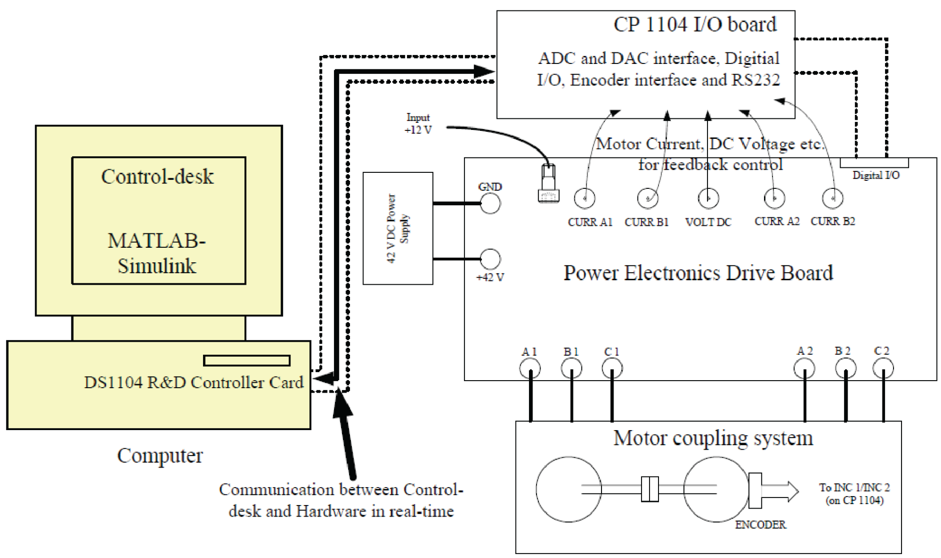

Figure 2: Laboratory experimental apparatus

In this experiment we will be commanding the DC drive motor in open-loop and observing the resulting speed based on the output of an encoder attached to the motor shaft. Figure 3 illustrates the elements involved in commanding the motor and reading the resulting velocity. For the purposes of identification, we will assume the simplified system shown in Figure 4. This simplified system agrees with the linear models we developed from first principles. This simplified representation of the actual system is a reasonable approximation for a couple of reasons. For one, the frequency of the pulse train is fast compared to the bandwidth of the physical motor, therefore, the motor acts like a low pass filter. Additionally, the dynamics of the encoder and the subsequent filter are fast compared to the dynamics of the motor and hence can be approximated as a static gain.

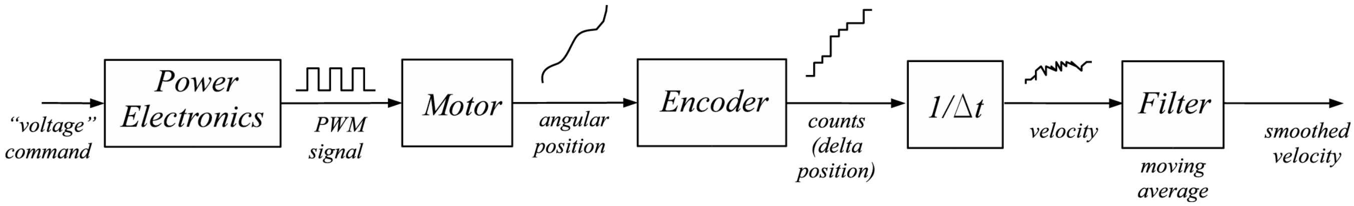

Figure 3: Block diagram of the laboratory set-up

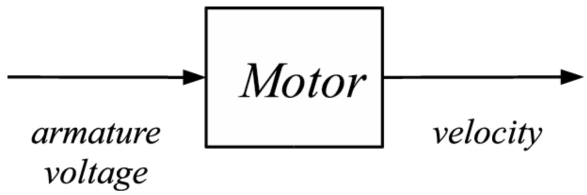

Figure 4: Simplified representation of the laboratory set-up

2.2 Start up

1. *Connect Hardware* - For this experiment, connect the positive terminal of the DC-generator armature to connector **PHASE A1** on the power electronics board and the negative terminal to connector **PHASE B1**. Note, we will use the DC "generator" as our drive motor. Next connect **CURR.A1** (motor armature current) of the power electronics board to the analog input channel **ADCH7** of the I/O board. Also attach the encoder output (mounted on the DC-generator) to the **INC1 9-Pin D SUB** connector of the I/O board. Power is supplied to the electronics board from a variable DC power supply. Set the DC power supply to provide 42 V and set the current limit sufficiently high to source what is needed by the motor. Ask for instructor permission before switching on the output of the power supply and switching on the **Analog Power** on the power electronics board.

2. *Simulink set-up* - Start the MATLAB software (version R2007a) on the desktop computer. When prompted identify that the model of dSPACE board being used as 1104. Within MATLAB, set the working directory to the following.

   C:/Documents and Settings/Student/Desktop/5020LAB1/expPartA

   Once in this directory, open the Simulink model **expPartA.mdl**. Also set the global variables for DC voltage being supplied and sample time within MATLAB. This is done by typing `Vd = 42` and `Ts = 0.0001` at the MATLAB command line. Explore the Simulink model to get a sense of what is being done in the program. Note the type of filtering that is performed on the current and speed measurements.

3. *Build the executable files* - Pressing **CTRL+B** will build the control system real-time executable code. Specifically, the C-code that will run on the controller board is generated

by Simulink, while a second .sdf “variables file” to be used by the dSPACE ControlDesk software is also generated.

4. *Control interface set-up* - Start the dSPACE ControlDesk software on the desktop computer. Open the .sdf file containing the variables needed by the control software by clicking **File>Open Variable File**. This file can be found in the same working directory used above and is entitled **exp parta.sdf**. Next open the layout file **expPartA.lay** from the same directory by clicking **File>Open**. Click the **PLAY** button (green arrow) to begin the control program, unless already selected, then switch to **Animation mode** as shown in the Figure 5 below. Gently experiment with the controls to command the drive motor. Observe the resulting angular speed and armature current. Keep the voltage applied to the motor below 30 V. In order to stop the system, revert the system to **Edit Mode** and click the **STOP** button (red square).

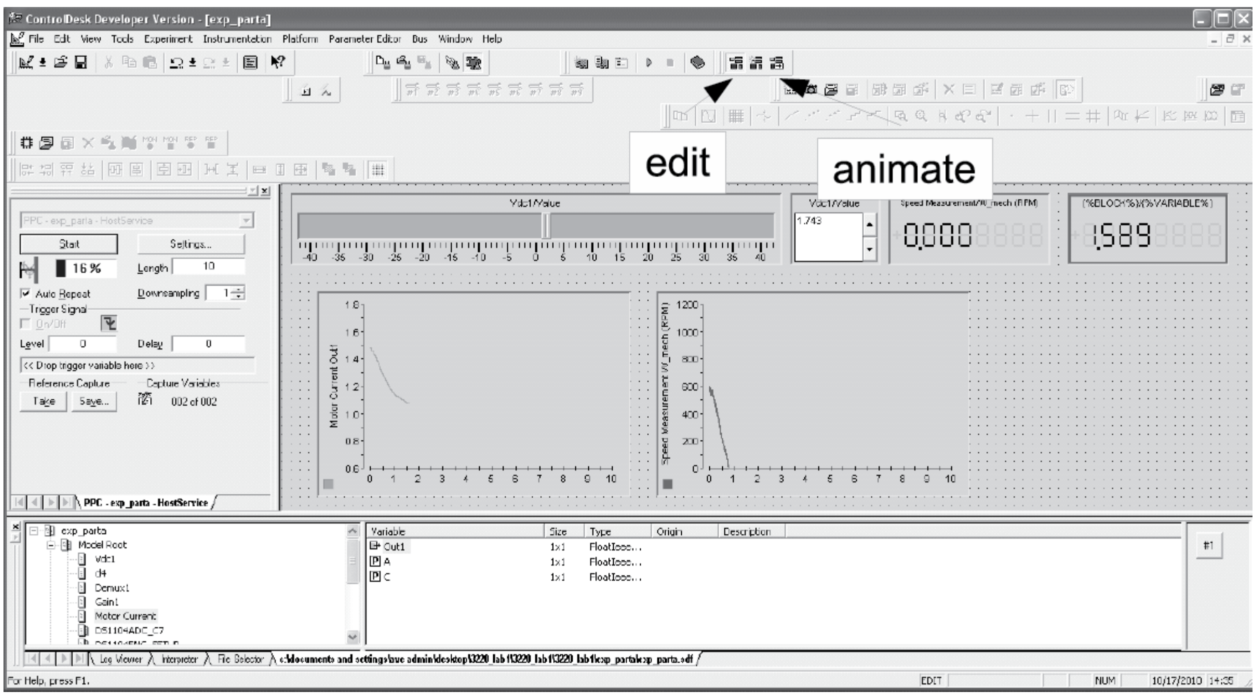

Figure 5: Various control buttons for starting and stopping the system

Furthermore, in order to capture this dynamic data within the ControlDesk environment, it is necessary to open the Capture Settings Window which can be opened by clicking **View>Control bars>Capture Settings Window**. Once the window is open, set the time **Length** to 10 seconds. The Capture Settings Window may initially open collapsed against the right side of the ControlDesk.

2.3 Friction Estimation

From inspection of Equation (2) describing the mechanical aspects of the motor, it can be seen that a model for the friction in the motor can be determined by observing the steady-state operation of the motor at a range of speeds. In steady state $\ddot{\theta}_m(t) = 0$ and the applied torque T equals the friction torque $T_{friction}$. In class we have assumed a viscous model $T_{friction} = b\dot{\theta}_m$ since it is linear. Here we will employ a modified model that includes a "stiction" element, $T_{friction} = T_{stiction} + b\dot{\theta}_m$, in order to more accurately capture the behavior of the motor. Fill out the following table for a range of constant voltage inputs, making note that current relates to motor torque. Do not increase the commanded voltage in steps larger than 5 V. Also, do not command a voltage greater than 30 V.

e_a [V]	$\dot{\theta}_m$ [rpm]	$\dot{\theta}_m$ [rad/s]	i_a [A]	T [Nm]
3				
5				
10				
15				
20				
25				

Using the above tabulated data, generate a graph of motor speed versus torque. Based on a line fit to this graph, and Equation (2), estimate the parameters of the friction model $T_{stiction}$ and b. Since the drive motor is connected to the load motor in this process, you are actually estimating the friction in the entire system (both drive and load motor).

2.4 Inertia Estimation

Whereas the previous section of the experiment required examination of the motor in steady state, this portion of the experiment will require an examination of the dynamic response of the system. In this experiment the motor is commanded to some steady state velocity $\dot{\theta}_m$, then the supplied armature voltage is abruptly dropped to zero. Hence the armature current will go to zero and the resulting motor torque will also go to zero. Since we are interested in the transient behavior of the system in this case, you may wish to go back into the Simulink file used above and remove the filter applied to the current output. You will then need to re-build the executable code for real-time implementation. You do not need to remove the moving average filter on the motor speed, think about why.

Within the ControlDesk environment, gradually increase the command to the motor to 25 V. Once the motor reaches steady state, wait until the plotting window refreshes, then uncheck the box for the SHUTDOWN signal. The SHUTDOWN signal shuts down the PWM signals to the semiconductor switches. The armature current then freewheels through the diodes against the motor's back emf, supply voltage and the armature resistance, and drops to 0 within milliseconds.

Stop the program when the graphs appear to have approached steady state. This data is then saved by clicking the **Save** button in the Capture Settings Window. This process generates a .mat file which can then be loaded into MATLAB and plotted. The data can be accessed within MATLAB from a variable of the same name as the .mat file. For example, if the filename was **inertia.mat**, then **inertia.X.Data** contains the time vector, **inertia.Y(1).Data** contains the current data, and **inertia.Y(2).Data** contains the speed data.

Again using the mechanical motor model given in Equation (2), the inertia of the motor can be solved for based on a given instant of data. You may treat the armature current as dropping to zero almost instantaneously at the point that the motor is shutdown. The graph of the current will not reflect this instantaneous drop unless you have removed the low-pass filter on the current signal within Simulink. Note that a derivative can be estimated graphically from the slope of a line tangent to the curve at a given point in time. Do not forget to include the stiction term in your friction model. At the instant just prior to the SHUTDOWN signal being sent, the motor is operating at a constant speed and the applied motor torque T is exactly balanced by the friction torque $T_{friction}$.

2.5 Inductance Estimation

The inductance of the motor can be identified from the data shown in Figure 6. This data represents the response of the DC motor's armature current for a 4 V step in the armature voltage with the motor rotor blocked. The inductance can be solved for using Equation (1) which models the electrical aspects of the motor. The purpose of blocking the rotor so that it doesn't move is that it results in there being no back emf since $\dot{\theta}_m$ is zero. Notice that when the back emf is taken out of Equation (1) the expression models a first-order system. The graph shown in Figure 6 approximately has the character of the first-order step response we are familiar with from class. Note that this current data is not filtered. The .mat file associated with this graph will be made available on the course Blackboard site.

3 Part B: Black-Box Model Derivation

The second approach to model derivation that we have learned about is a black-box approach. In this particular instance, we will command a step input of armature voltage and observe the corresponding speed response. We will then attempt to identify the response as being characteristic of one of our standard types and will estimate the corresponding parameters. For this experiment, you are to specifically capture data for a step in voltage from 0 to 4 V, as well as for a step from 10 to 14 V.

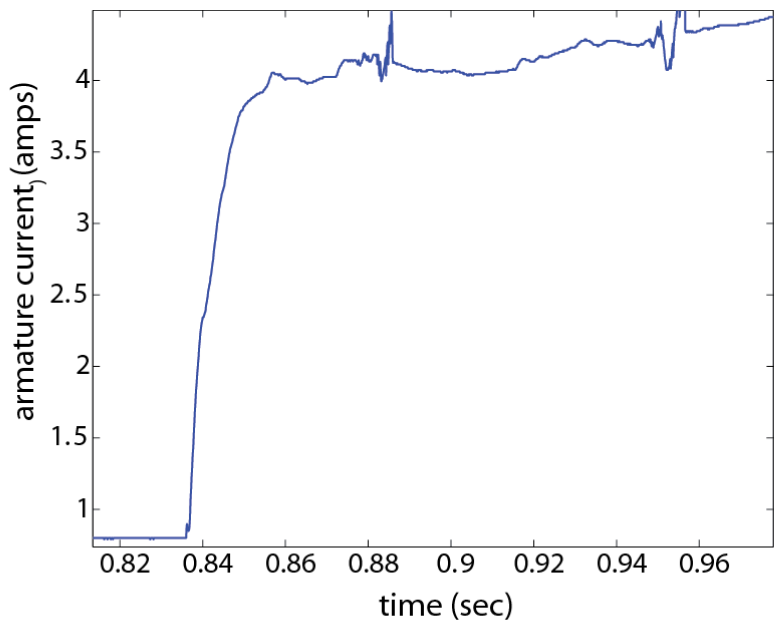

Figure 6: Response of armature current (amps) to a 4 V step input with the rotor blocked

4 Lab Assignment

You do not need to do a formal report for this laboratory. Instead, you need to answer the following questions in a clear manner with sufficient work shown to support your conclusions.

1. (30 points) *First-Principles Model* - Identify the unknown parameters needed to fill out the first-principles model given in Equations (1) and (2). Specifically, you need to estimate the armature inductance L_a, the motor inertia J, and the characteristics of the friction model b and $T_{stiction}$. Show any work you did to arrive at your estimates, including information taken from graphs.

2. (15 points) *Black-Box Models* - Identify a transfer function model for each of the two sets of step response data you took. Note that the input to the motor was a 4 V step, not a unit step. Also, for the step with an initial condition of 10 V, you can look at just the change in input and output (like a linear approximation) since transfer functions do not include initial conditions. Be careful of units.

3. (45 points) *Simulation* - Simulate each of your models and compare them to the step response data you took from the actual motor (0 to 4 V and 10 to 14 V). For the first-principles model, consider how you will include the stiction element of the friction model as well as the initial conditions. Provide pictures of your models and the resulting graphs. Also discuss how well each of your models match the actual data. There are bound to be experimental errors in your data, therefore, your models may not match the actual data very well. Attempt to use your

knowledge from class regarding time response to pinpoint what specific parameters might be in error to explain any mismatch that you observe.

4. (10 points) *Discussion of the Modeling Formalisms* - Discuss the advantages and disadvantages of each of the modeling approaches employed. Use the data and your experiences from the laboratory activity to support your arguments.

Note, you are to do this assignment in groups of two or three and the assignment is due Friday, July 5th. Leave enough time to ask any necessary questions (and to accomodate any travel plans you may have for the 4th of July). For your reference, I will post the Simulink file from lab on the Blackboard site, but note that you will not be able to run them because you do not have access to the physical hardware.