ENGR 4220/5220: Control Systems I

Professor Hill

University of Detroit Mercy, Winter 2014

Homework #7

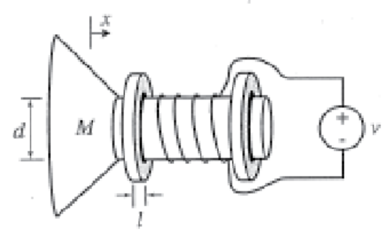
Assigned: February 13, 2014

Due: February 20, 2014

Read Sections 6-5 and 7-4 of the book.

Recommended example problems: A-6-18 and A-7-14

1. (20 points) The system given below is a loudspeaker. The speaker works by application of a voltage $v(t)$ to a voice coil that electromagnetically drives the plunger of mass M. The coil has inductance L and resistance R. The force generated by the voice coil motor is proportional to the current through the electrical circuit, $F = Ki$. Additionally, the system generates a back emf that is proportional to the velocity of the plunger, $v_b = K_b\dot{x}$. The motion of the plunger is also opposed by friction in a circular bearing. The friction is modeled as being proportional to the speed of the plunger with a friction coefficient that is a function of the surface area of the bearing and the viscosity of the oil in the bearing, that is, $b = \mu d\pi l$.

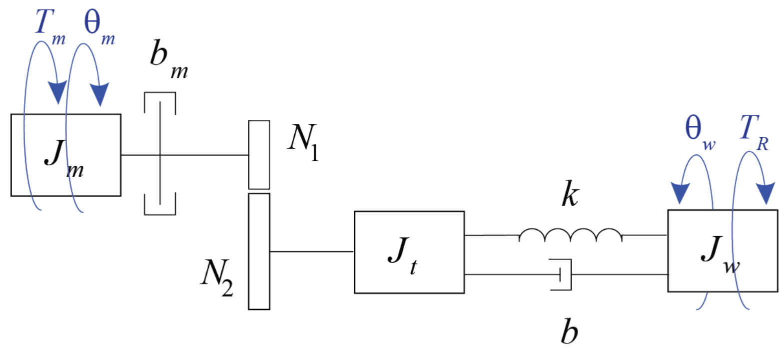


   (a) Derive a system of differential equations to model the given loudspeaker system. Note you will need an equation for the electrical portion of the speaker as well as the mechanical portion.

   (b) Find a transfer function model for this system regarding the voltage $v(t)$ as the input and the position of the plunger $x(t)$ as the output. Assume units.

   (c) Modify your answer from part (b) to generate a transfer function for an input of voltage $v(t)$ and an output of plunger velocity $\dot{x}(t)$. Again assume units.

2. (25 points) Consider the figure shown below that models the driveline of an electric vehicle.

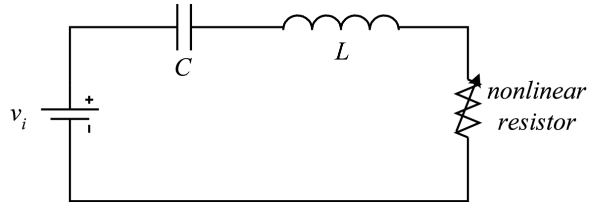


The motor generates the torque T_m and has a rotational inertia of J_m and experiences a frictional torque that opposes its motion and is modeled as being proportional to the motor's speed, that is, $T_{friction} = b_m\dot{\theta}_m$. The output shaft of the motor is considered rigid and rotates with angular displacement θ_m. The motor torque is transmitted through a transmission to another shaft that is attached to the wheels. The transmission is modeled as having a rotational inertia of J_t and the wheels as having inertia J_w. The shaft connecting the output of the transmission to the wheels is considered flexible and is modeled as having a stiffness k and damping b. The wheel moves with angular displacement θ_w and experiences a reaction torque from the road T_R.

(a) Draw the free-body diagrams for each of the inertias in the above and determine their associated equations of motion. Do not forget the reaction torques at the input and output of the transmission.

(b) Use your knowledge about gear trains to rewrite your equations from Part (a) in terms of only the motor torque T_m and road torque T_R and in terms of only the displacement of the motor θ_m and the wheels θ_w.

3. (20 points) Consider the electrical circuit shown in the figure below. It has a voltage source v_i, a linear inductor with inductance L, a linear capacitor with capacitance C, and a nonlinear resistor. The voltage across the resistor is an exponential function of the current passing through it, that is, $v = ke^i$.



(a) Write out the nonlinear equation describing the above circuit. Rewrite the equation in terms of the charge on the capacitor q where $i = \dot{q}$ if you didn't do so initially.

(b) For a nominal input voltage $\overline{v}_i$, find the equilibrium charge $\overline{q}$ stored on the capacitor.

(c) Linearize the exponential function for the voltage across the resistor $f(\dot{q}) = ke^{\dot{q}}$ about the equilibrium $\overline{q}$ you found in part (b).

(d) Now linearize the differential equation you found in part (a) about the equilibrium $(\overline{v}_i, \overline{q})$ and express the linearized equation in terms of small increments of voltage and charge $\delta v_i = v_i - \overline{v}_i$ and $\delta q = q - \overline{q}$.

4. (20 points) It is tempting to model the exponential growth of a bacteria in a colony as

$$\dot{x} = ax$$

where $x(t)$ represents the amount of bacteria in the colony and $a > 0$ is a constant defining the exponential growth rate. However, as the colony grows (x increases), food becomes scarce, and the growth rate slows. This can be modeled using the so-called *logistic equation*:

$$\dot{x} = ax(1 - \frac{x}{M})$$

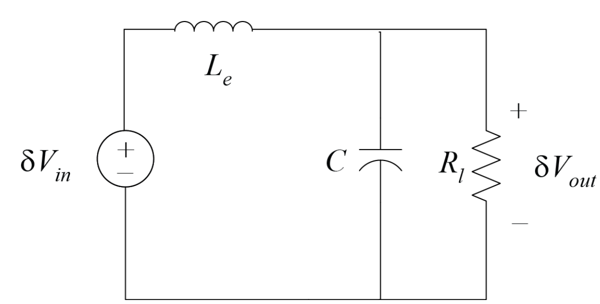
This is a simple, but nonlinear, model for the growth of bacteria in a colony with a finite food source. Note that $x_o = 0$ is an equilibrium of this system.

(a) Find another (nonzero) equilibrium $\bar{x} \neq 0$ solution to the logistic equation.

(b) Linearize the logistic equation around the equilibrium point you found in Part (a). Express your solution as a differential equation in terms of δx.

(c) Is the linearized system you found in Part (b) stable? Why or why not? What do you think the stability of the linearized system means for the stability of the real nonlinear system?

5. (15 points) If the switching frequency of a boost converter is high enough, an average linear model of the converter is often used. One such average model is shown below where the equivalent inductance L_e is related to the actual inductance L by the relation $L_e = L/(1-D)^2$ where D is the duty cycle of the switching and δV_{in} and δV_{out} are incremental changes in voltage from some nominal input and output, respectively. The capacitor C is included to filter the ripple in the output voltage, and the load is modeled as a simple resistance R_l.



It is often desirable to consider the change in duty cycle δD to be the input to the converter. For this input and an output of δV_{out}, the transfer function of the linearized system is:

$$\frac{\Delta V_{out}(s)}{\Delta D(s)} = \frac{V_{in}}{(1-D)^2} \frac{R_l - sLe}{R_l C L_e s^2 + s L_e + R_l}$$

(a) Consider the transfer function of the linearized converter shown above with $R_l = 100\Omega$, $C = 10 \times 10^{-6}$ F, and $L = 200 \times 10^{-6}$ H. For a nominal duty cycle of $\overline{D} = 0.5$ and a nominal supply voltage of $V_{in} = 100$ V, the nominal output voltage will be boosted to $\overline{V}_{out} = \frac{1}{1-\overline{D}} V_{in} = 200$ V. Simulate this linear model for a step in duty cycle from 0.5 to 0.51 and a step in duty cycle from 0.5 to 0.6. Run the simulation for 0.02 seconds and let the step occur at 0.01 seconds.

(b) Now download the fully nonlinear Simulink model of the boost converter from Blackboard. By right-clicking on the converter block you can look under the mask to see the full model. Simulate this model for a step in duty cycle from 0.5 to 0.51 and a step in duty cycle from 0.5 to 0.6.

(c) Compare the results for the linear and nonlinear model for the two different step inputs and attempt to explain why the models match better in one case than the other.

(d) Describe the tradeoffs associated with using the linear and nonlinear models.