ENGR 4220/5220: Control Systems

Professor Hill

University of Detroit Mercy, Winter 2014

Homework #8

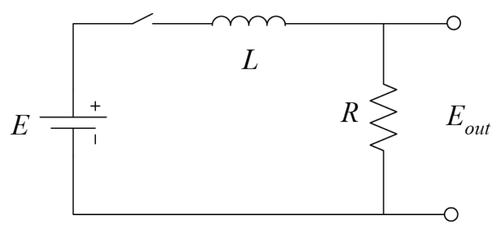
Assigned: February 27, 2014

Due: March 6, 2014

Read Sections 8-1 to 8-3 of the book.

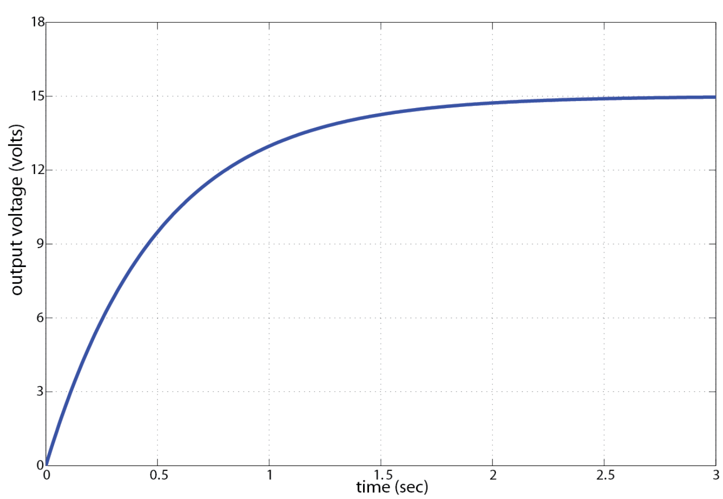
Recommended example problems: A-8-1, A-8-2, A-8-3, A-8-7, and A-8-8

1. (20 points) Consider the following electrical circuit.



   (a) Find a transfer function model for this circuit where the input is the voltage E and the output is the voltage across the resistor E_{out}.

   (b) For a constant voltage source of E = 15 Volts, if there is no current in the circuit and the switch is closed, the voltage across the resistor E_{out} increases as shown in the following figure. Based on this graph, determine a relationship between the resistor R and inductor L in the circuit.



   (c) How could you change the parameters of the given circuit to make it respond faster?

2. (35 points) An empirical model of a DC motor is derived as follows where the applied voltage $v(t)$ is the input and the motor speed $\omega(t)$ is the output.

$$\frac{\Omega(s)}{V(s)} = \frac{100}{s+10} \; \frac{rad}{V \cdot s}$$

(a) Consider that this motor begins from rest and is given a voltage input that steps from 0 Volts to 5 Volts and then steps back to 0 Volts after τ seconds where τ is the time constant of the motor. Sketch a graph of the motor's speed in response to this input. Make sure to identify the motor's speed at the time of the switch back to 0 Volts.

(b) Estimate the speed of the motor 2τ seconds after the initial step in voltage from 0 Volts to 5 Volts (τ seconds after the switch back to 0 Volts).

(c) Often DC motors are controlled by what is referred to as Pulse Width Modulation. In this approach the motor is either fed a constant positive voltage or no voltage at all. In this regard, the speed of the motor is affected by the percentage of time the motor is "ON", in other words, by changing the width of the commanded pulse. Draw a hypothetical graph of what the motor speed might look like for a series of pulse commands.

(d) Your graph in Part (c) likely had ripple in it. How might you reduce this ripple in motor speed?

3. (25 points) Consider that a battery in an electric vehicle is installed in a rectangular package such that all but two sides of the package are well insulated. It is given that the battery package generates heat at rate q and has some thermal capacity C. Also, the process of the heat escaping through the two uninsulated walls is described by a thermal resistance R that is a function of the wall materials and their geometry. Therefore, the equation describing the temperature of the battery temperature T_B is as follows, where T_O is the ambient temperature outside of the package.

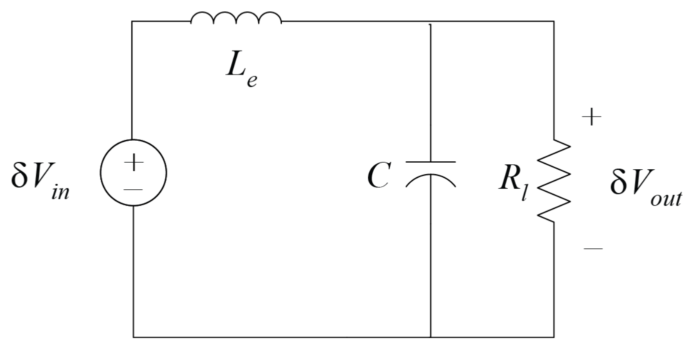
$$\dot{T}_B = \frac{1}{C}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)(T_O - T_B) + \frac{1}{C}q$$

(a) Rewrite the given differential equation in terms of the temperature difference $\Delta T = T_B - T_O$ where it can be assumed that the ambient temperature T_O is constant.

(b) Find the transfer function $\Delta T(s)/Q(s)$.

(c) Is this system BIBO stable? Does your answer make physical sense to you? Explain.

4. (20 points) If the switching frequency of a boost converter is high enough, an average linear model of the converter is often used. One such average model is shown below where the equivalent inductance L_e is related to the actual inductance L by the relation $L_e = L/(1-D)^2$ where D is the duty cycle of the switching and δV_{in} and δV_{out} are incremental changes in voltage from some nominal input and output, respectively. The capacitor C is included to filter the ripple in the output voltage, and the load is modeled as a simple resistance R_l.



The transfer function for the above converter for an input of δV_{in} and an output of δV_{out} is

$$\frac{\Delta V_{out}(s)}{\Delta V_{in}(s)} = \frac{R_l}{R_l C L_e s^2 + s L_e + R_l}$$

(a) Let $R_l = 100\Omega$, $C = 10 \times 10^{-6}$ F, $L = 200 \times 10^{-6}$ H and $D = 0.50$. For these parameters, estimate the peak time t_p, the settle time t_s, and the maximum percent overshoot M_p for a unit step change in the input voltage.

(b) Sketch a rough graph of the response of δV_{out} for unit step input, that is, $\delta V_{in} = 1(t)$.