ENGR 4220/5220: Control Systems

Professor Hill

University of Detroit Mercy, Winter 2014

Homework #13
Assigned: April 10, 2014
Due: April 17, 2014

Read section 11-6 of the book.
Recommended example problems: A-11-6, A-11-7, A-11-11, A-11-12, A-11-13

1. (35 points) You wish to design a controller for a DC motor position control system.

   (a) Consider the following transfer function consisting of a PD compensator in series with the DC motor.

   $$G_{PD}(s) = 5(s+4) \cdot \frac{1}{s(s+2)}$$

   Sketch a straight-line approximation of the Bode plot of the above transfer function by hand. Check your result with the MATLAB command **bode**.

   (b) Now consider the DC motor in series with what is referred to as a lead compensator.

   $$G_{lead}(s) = 150\frac{s+4}{s+30} \cdot \frac{1}{s(s+2)}$$

   Again sketch a straight-line approximation of the Bode plot and check with MATLAB.

   (c) Discuss what the magnitude plots from Part (a) and Part (b) mean for how the system will respond differently under lead compensation as compared to PD control.

   (d) Determine the stability margins of the system given in Part (b) based on the MATLAB generated Bode plot. Is it possible to achieve a phase margin of 45 degrees simply by adding a gain K to the compensator? Explain.

2. (25 points) Consider the following model and parameters of an armature-controlled DC motor with armature voltage as the input and angular speed as the output:

$$\frac{\Omega(s)}{E_a(s)} = \frac{K}{(sL_a + R_a)(Js + b) + KK_b}$$

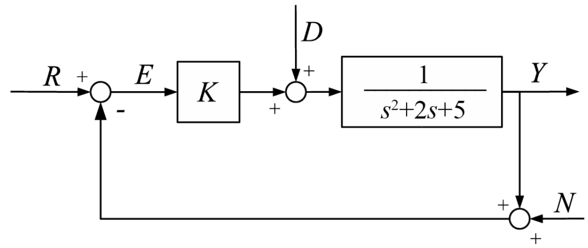
R_a = 0.45 Ω	b = 0.028 Nm/rad/s	J = 1.13×10^{-2} Nm/rad/s^2
L_a = 1×10^{-1} H	K = 0.067 Nm/A	K_b = 0.067 V/rad/s

(a) Use MATLAB to plot the Bode diagram of the motor in series with a 100-gain amplifier. Identify the gain crossover frequency ω_c for this system.

(b) The speed of a DC motor is often controlled by pulse-width modulation (PWM) where the motor is fed a square wave alternating between full on and full off. The speed of the motor is affected by altering the percent of time that the wave is high, this is called the duty cycle. Simulate the response of the motor transfer function for a series of square waves (50% duty cycle) with frequencies $0.1\omega_c$, ω_c, and $10\omega_c$. You may use the model posted on the Blackboard site. Explain what you see.

(c) A step input of what magnitude will provide a response that most closely resembles your simulation results? Explain how you would employ PWM control to achieve the same behavior as if you commanded the motor with a continuous signal.

3. (40 points) Consider the feedback system shown below with proportional controller K.



(a) Plot the Bode diagram of the open-loop system with $K = 5$ and $K = 50$. Discuss the anticipated effect of increasing the proportional gain K on the closed-loop system's response based on the Bode diagrams. Consider speed, overshoot, steady-state effort, and required control effort.

(b) Calculate the closed-loop transfer function from R to Y for $K = 5$ and $K = 50$. Based on these Bode diagrams, would you change anything about your prediction from Part (a)? Discuss the advantages and disadvantages of employing the Bode diagram of the open-loop and closed-loop system for analyzing and designing the closed-loop system.

(c) Simulate the closed-loop system for a unit step reference without any noise and without a disturbance. Plot the output y versus time and the control effort u versus time. Do the results agree with what you predicted in Parts (a) and (b)?

(d) Calculate the closed-loop transfer function from D to Y for $K = 5$ and $K = 50$. Generate the Bode magnitude plots for both cases. Based on these Bode diagrams, how would you expect the system's ability to reject a step disturbance would change?

(e) Simulate the closed-loop system for a unit step reference and a unit step disturbance that occurs at t = 5 seconds. Plot the output y versus time for $K = 5$ and $K = 50$. Describe what you see. Is it what you expected?

(f) Calculate the closed-loop transfer function from N to Y for $K = 5$ and $K = 50$. Generate the Bode magnitude plots for both cases. Based on these Bode diagrams, how would you expect the system's ability to reject a sinusoidal noise input of frequency 5 rad/sec would change?

(g) Simulate the closed-loop system for a unit step reference and a sinusoidal noise input of frequency 5 rad/sec and amplitude 0.1. Plot the output y versus time for $K = 5$ and $K = 50$. Describe what you see. Is it what you expected?