ENGR 4220/5220: Control Systems

Professor Hill

University of Detroit Mercy, Winter 2014

Homework #9

Assigned: March 13, 2014

Due: March 20, 2014

Read Sections 8-3, 8-4, 10-1, 10-2, and 10-5 of the book.

Recommended example problems: A-8-7, A-8-8, A-10-1, A-10-2, A-10-5, and A-10-6

1. (20 points) Problem B-8-7, page 426.

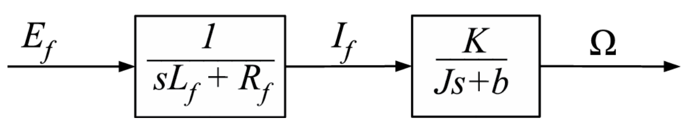
2. (20 points) Consider the following transfer function of a *field-controlled* DC motor with field voltage as the input and angular speed as the output:

$$\frac{\Omega(s)}{E_f(s)} = \frac{K}{(sL_f + R_f)(Js + b)}$$

Additionally assume the following motor parameters.

R_f	$= 2\ \Omega$	b	$= 4 \times 10^{-4}$ Nm/rad/s
L_f	$= 5.2 \times 10^{-6}$ H	K	$= 0.1$ Nm/A
J	$= 152 \times 10^{-6}$ kg-m^2		

The block diagram shown models the given transfer function.



(a) Determine the DC gain and pole for the following transfer function representing the electrical dynamics of the DC motor:

$$\frac{I_f(s)}{E_f(s)} = \frac{1}{sL_f + R_f}$$

Replace the corresponding dynamic block in the given block diagram by the static gain you just found. What is the resulting reduced-order transfer function for an input of $E_f(s)$ and an output of $\Omega(s)$?

(b) Repeat Part (a), but with the original electrical dynamics and this time with the mechanical dynamics approximated as a static gain.

(c) Using the MATLAB command **step**, determine the responses of the original transfer function and the responses of the reduced-order transfer functions from Part (a) and Part (b) for a unit step input. Compare the three responses and attempt to explain any differences.

3. (20 points) Recall from class that the transfer function of an armature-controlled DC motor with armature voltage as the input and angular speed as the output is the following:

$$\frac{\Omega(s)}{E_a(s)} = \frac{K}{(sL_a + R_a)(Js + b) + KK_b}$$

(a) For the following set of motor parameters, determine the poles of the above transfer function.

R_a = 0.2 Ω
L_a = 1×10^{-6} H
J = 1×10^{-5} lbf-ft-s^2
b = 4×10^{-3} lbf-ft/rad/s
K = 6×10^{-5} lbf-ft/A
K_b = 5.5×10^{-2} V-s/rad

(b) Determine a reduced-order approximation of the given transfer function.

(c) Using the MATLAB command **step**, plot the unit step response of the original transfer function together with the step response of the reduced order transfer function. How well does the approximation match the original transfer function? What determines how well the reduced transfer function approximates the original transfer function?

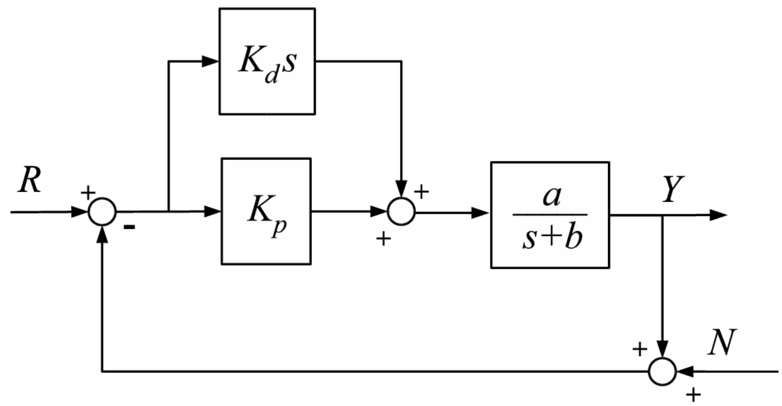
4. (15 points) Consider the canonical second-order transfer function $\frac{Y(s)}{R(s)} = \frac{25}{s^2+2s+25}$.

(a) Analytically calculate the peak time t_p, 2% settle time t_s, and percent overshoot M_p for a a unit step response. Then use the MATLAB command **step** to plot the unit step response for this system. From the resulting graph, verify your analytical results.

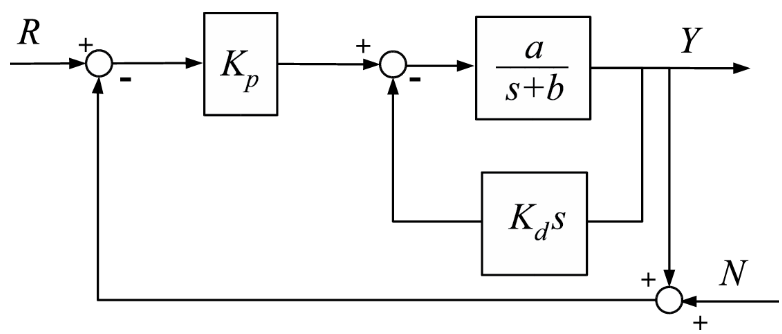
(b) How would your results be affected if the numerator was set to (i) 1 (ii) $25 + 4s$? No calculations are needed for Part (b).

5. (25 points)

(a) Reduce the block diagram shown below to find the transfer function for an input of R and an output of Y.



(b) The block diagram in Part (a) has a controller with derivative action in the forward path. This architecture can cause problems if there are rapid changes in the reference. One solution is to move the derivative term to the feedback path as shown in the block diagram below. Reduce this block diagram to find the transfer function for an input of R and an output of Y and compare the result to the transfer function you found in Part (a).



(c) Considering the block diagram from Part (b), find the transfer function for an input of N and an output of Y.

(d) If the system shown in Part (b) experiences inputs of N and R simultaneously, what is the total output $Y(s)$?