Sample Final

ELEE 4700/5700, Controls II

Professor Hill

University of Detroit Mercy, Fall 2012

Name: ______________________________

Date: ______________________________

Scores:	Problem 1	________	35 points
	Problem 2	________	20 points
	Problem 3	________	20 points
	Problem 4	________	25 points
	total	________	100 points

Please write your answers in the spaces provided on the exam. If you use the backs of the pages or extra sheets, provide a note and/or arrows to point out the location of your solution.

1. (35 points) Consider the system modeled by the following set of differential equations.

$$\begin{aligned}\dot{a} &= b - 2a \\ \dot{b} &= -2a + 3c\end{aligned}$$

(a) (2 points) Find the A, B, C, and D matrices for the above equations, assuming a and b are the state variables, c is the input, and a is the output.

(b) (5 points) Is the open-loop system stable? Is the system controllable? Find the open-loop poles.

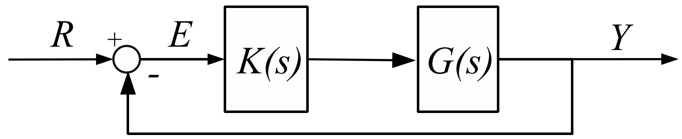
(c) (3 points) Convert the system model into transfer function form.

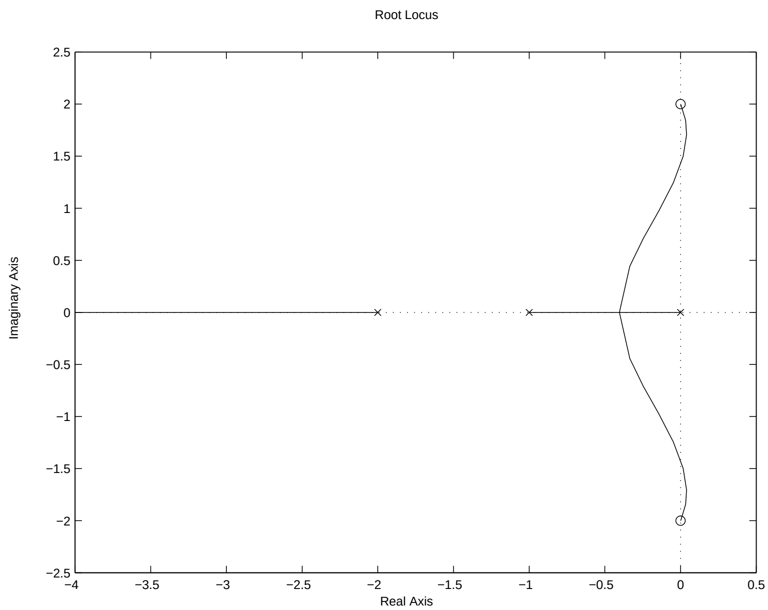
(d) (2 points) What would the C matrix be for the open-loop system if both a and b were outputs?

(e) (12 points) Determine the state-gain matrix $\mathbf{K}$ such that the state-feedback control law $u = -\mathbf{K}\mathbf{x}$ places the closed-loop poles at $-1 \pm 3j$. Note that the control law should be written in terms of the coordinate frame of Part (a).

(f) (11 points) Now consider that the state-feedback law $u = r - \mathbf{K}\mathbf{x}$ is employed. Find the closed-loop transfer function $Y(s)/R(s)$. What is the steady-state output if $r(t)$ is a unit step? What is the steady-state error?

2. (20 points) Consider the following unity-feedback control system where the root locus of the plant $G(s)$ is shown below.





(a) (3 points) Estimate the minimum 2% settling time that can be achieved using a proportional controller $K(s) = K_P$.

(b) (3 points) Using the pole-zero locations from the root locus, estimate the transfer function for the plant including an unknown constant.

(c) (5 points) A proportional-derivative compensator, written in the form $K(s) = K(s+z)$, is to be designed such that the dominant closed-loop poles are positioned at $-1 \pm j$. Use the root-locus approach to place the zero z.

(d) (6 points) Generate an approximate sketch of the root locus with the addition of the proportional-derivative controller. You do not need to find the break-out/break-in points, or the departure/arrival angles.

(e) (3 points) Which of the following controllers offers the same advantages of a PD controller, without amplifying high frequency noise?

(a) proportional control

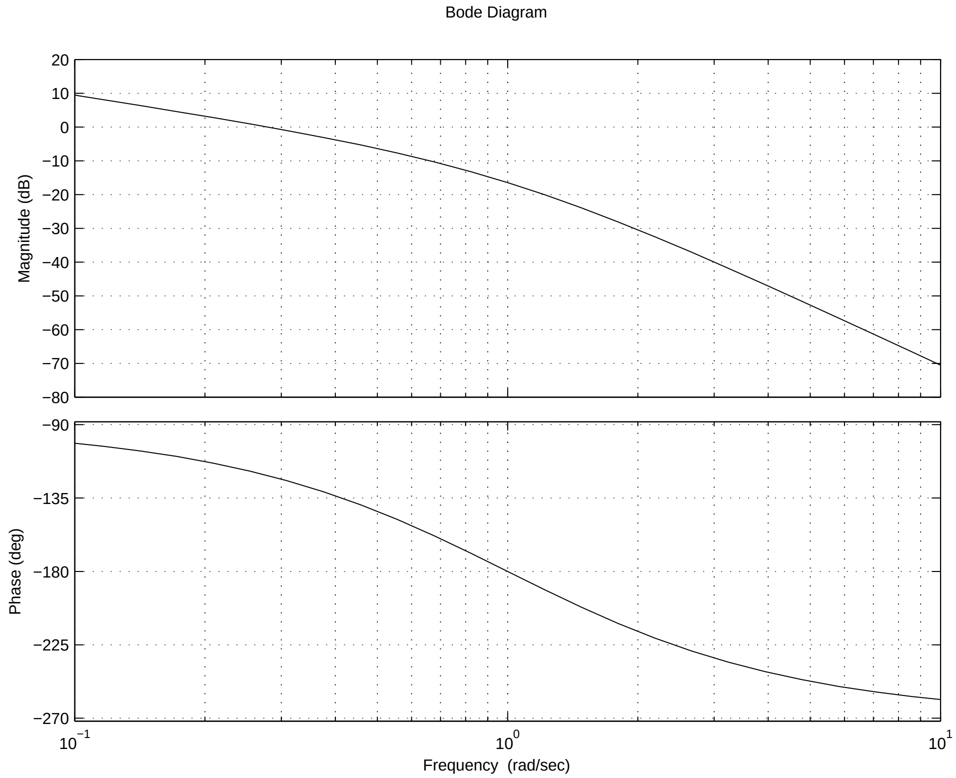
(b) proportional-integral control

(c) lag control

(d) lead control

(e) observer state-feedback control

3. (20 points) Consider the Bode plot of an open-loop system given below.



(a) (4 points) Find the gain and phase margins for this system in unity feedback.

(b) (3 points) If a proportional controller is used for this system in unity feedback, for what range of positive gains $K_P \geq 0$ is the closed-loop system stable?

(c) (8 points) Design a proportional controller for this plant under unity-feedback control so that the phase margin is ≥ 45 degrees, the gain margin is ≥ 5 db, and the steady-state error is as small as possible. What are the resulting stability margins?

(d) (5 points) Suppose that complex numerical computations (such as filtering) are required to determine the output y from the sensor measurements and these calculations take 0.5 seconds. How does this change the phase margin you found in Part(a) for the original system?

4. Consider the plant

$$G(s) = \frac{s+1}{s^3 + 6s^2 + 11s + 6}$$

(a) (5 points) Find a set of closed-loop poles to produce a step response with 10% overshoot and a peak time of 2 seconds.

(b) (3 points) Why might the closed-loop poles found in Part (a) not achieve the desired overshoot and peak time?

(c) (3 points) Determine the state-space matrices for the above plant in observable canonical form.

(d) (10 points) It turns out that not all of the state variables can be measured. Design a full-state observer for the system in observable canonical form. How did you choose where to place the observer poles?

(e) (3 points) Which of the following choices describe the limitations of an observer state-feedback approach to control? You may select more than one answer.

(a) the addition of an observer leads to a high bandwidth controller that passes high frequency noise

(b) the resulting observer-controller is n^{th} order, which tends to be larger than those controllers produced by root-locus or frequency-domain approaches

(c) observer state-feedback control cannot handle MIMO systems

(d) observer state-feedback control is difficult to design and implement using computers

(e) with observer state-feedback control, the effect of zeros are not accounted for in the design

(f) (3 points) Which of the following statements is false?

(a) the observer and the state-feedback controller can be designed separately

(b) the poles of the resulting closed-loop system are equal to the observer poles and controller poles taken together

(c) if the closed-loop system is stable, then the observer-controller must also be stable

(d) if a transfer function has a pole-zero cancelation, the system cannot be both controllable and observable

(e) for a given system, there are multiple possible state-space representations