BIOE 302 - Modeling Human Physiology: Cardiovascular Model

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Table of Contents

Simulink Model

The Simulink models presented simulate the cardiovascular systemic arterial pressure changes over time, and left ventricular pressure changes over time. It is adapted from Hoppensteadt and Peskin (2002). The model is based off of several conservation-based differential equations relating changes in volume, compliance and pressure over time.

The cardiovascular system is modeled at two different levels, 1) examining only a compliant systemic artery and 2) including the left ventricle along with the systemic artery. The first model, Cardio_SA_only.mdl, exclusively deals with the systemic arterial pressure changes over time, using a triangular waveform signal to represent blood flow into the aorta. All initialization values with units are preloaded into the Simulink file through the callback function in model properties, and are also available in the parameters_SA_only.m file.

The second model, Cardio_SA_LV.mdl, includes both the systemic arterial and left ventricular changes over time. The flow in this simulation model is driven by time-varying compliance changes of the left ventricle. All initialization values with units are preloaded into the Simulink file through the callback function in model properties, and are also available in the parameters_SA_LV.m file.

Cardiovascular Model – Systemic Arterial Only

Dynamics of Pulsatile Blood Flow

Blood vessels are characterized two properties: 1) Resistance: they provide some resistance to blood flowing through them and 2) Compliance: they distend due to changes in blood pressure. If we look at Figure 1, we have a blood vessel with an input flow (Q_1), output flow (Q_2), input pressure (P_1), output pressure (P_2), blood volume V and an atmospheric pressure (P_{ext}). If we want to find the volume, we would need to determine its relationship to blood flow and pressure. Unfortunately, because of the resistive and compliant nature of blood vessels, it is difficult to solve for the general case. It is best to take a simplifying scenario and make certain assumptions.

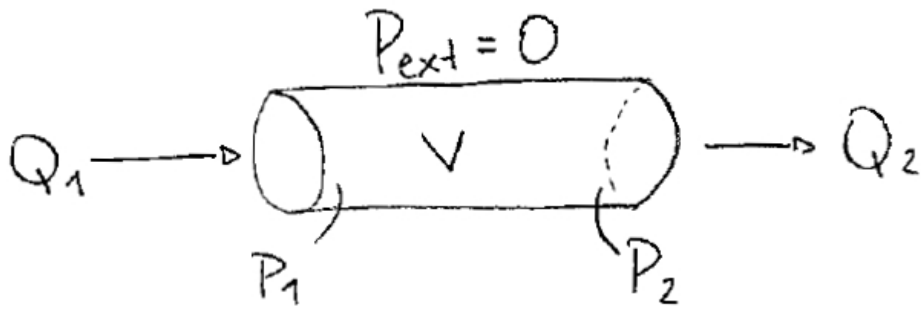

Figure 1: Blood vessel with input (Q_1), output (Q_2), volume (V), incoming pressure (P_1) and outgoing pressure (P_2) and atmospheric pressure (P_{ext})

First, lets assume that blood flow is at steady state where $Q_1 = Q_2 = Q$, making it a linear model for blood volume. Further, we are going to treat large, systemic arteries as elastic (or compliant) arteries where the resistance that they provide to blood flow is negligible but that they have elastic walls that store pressure. Smaller vessels of the microcirculation will be treated as resistive vessels that have little negligible compliance for holding pressure but provide significant resistance to blood flowing through them. For resistive vessels, blood flow is related to differences in pressure only, with the following relationship similar to Ohm's law:

$$Q = \frac{P_1 - P_2}{R}, \qquad (1)$$

and R is the resistance of the vessel.

Now let us consider the other vessel type, compliant arteries, the perfectly elastic vessel with no resistance to flow. Blood pressures are therefore equal at all points along the vessel and the volume is directly proportional to its pressure.

$$V = CP + V_d, \qquad (2)$$

C is a constant of proportionality and is the compliance of the vessel, and V_d is the dead volume of the vessel or the volume of blood in the vessel when the pressure is 0. Note that compliance relates changes in volume to changes in pressure. A large value for C will result in large volume changes in the vessel with small changes in pressure inside the vessel. For most of the derivations that follow, we will treat V_d as small and will ignore it.

Although we have treated separately two distinct types of vessels, real vessels however, exhibit both elastic and resistive properties. Fortunately, certain traits dominate over the other, and the assumptions of compliant vessels having negligible resistance and capillaries having negligible compliance results in a fairly accurate description of the overall system's function. Large arteries and veins tend to be highly compliant and have very low resistance. Arterioles tend to be muscular and behave as rigid vessels with resistive properties.

Our main assumption of constant pressures and flows used in deriving Equations (1-2) is far from realistic. Blood flow through vessels is not at steady state, as the heart pumps blood in bursts. These bursts or arterial pulses can be felt at the wrist or in brachial arteries with a sphygmomanometer. Typically, people will have a systolic pressure of 120 mmHg, and a diastolic pressure of 80 mmHg. The blood pressure therefore, varies over time due to its pulsatile nature. We can now begin to introduce the first of our dynamic cardiovascular models.

Large arteries are known to be compliant vessels, with low resistance and small pressure drops, serving to store pressure in the system during systole and to release it during diastole. We can modify Equation (2) by adding the time dependence on pressure and volume.

$$V(t) = V_d + CP(t), \qquad (3)$$

The elasticity of the peripheral arteries will respond to increased blood flow and pressure by expanding, therefore causing changes in blood volume. If we take a segment of an artery, we can determine the changes in volume over time by the difference in the input and output of its blood flow.

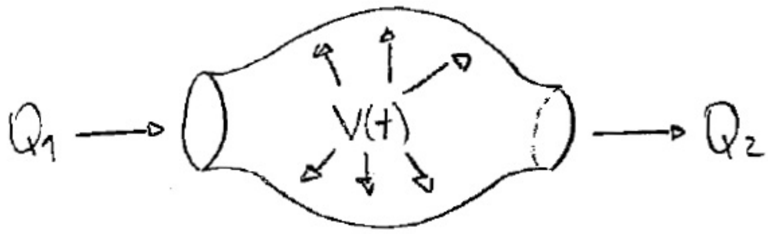

Figure 2: Compliant Vessel expanding due to blood flow.

$$\frac{dV}{dt} = Q_1 - Q_2 \qquad (4)$$

We can relate Equation (3) and (4) by taking the derivative of Equation (3) and matching it to Equation (4).

$$\frac{dV}{dt} = C\frac{dP}{dt} \qquad (5)$$

$$C\frac{dP}{dt} = Q_1 - Q_2 \qquad (6)$$

We now have the underlying concepts for our first dynamic cardiovascular model which includes the behavior of the primary systemic arteries. In our model, we will monitor changes in blood pressure over time in a systemic artery. We assume the artery to be a compliant vessel, with the inflow of blood (Q_1) coming from the aorta, and outflow of blood (Q_2) that flows out to the microcirculation in the arterioles and capillary beds, as systemic tissue circulation. We also assume the arterioles and capillaries can be treated as resistance vessels.

$$C_{SA}\frac{dP_{SA}}{dt} = Q_{Ao} - Q_S. \qquad (7)$$

The subscripts SA, denote systemic arterial, Ao, denote aortic and S systemic. Because we have assumed that the arterioles and capillaries are resistive vessels, Equation (1) can replace the Q_s term in Equation (7)

$$Q_s = \frac{P_{SA} - P_{SV}}{R_s}, \qquad (8)$$

where the incoming pressure is from the systemic artery, P_{SA}, and the outgoing pressure is equal to that of the systemic vein, P_{SV}. Because venous pressure is low, we will simplify our equations by neglecting the contribution to systemic flow from the venous pressure, simplifying Equation (8),

$$Q_s = \frac{P_{SA}}{R_S}. \qquad (9)$$

We can include Equation (9) into Equation (7) and end up with the governing differential equation for our model.

$$C_{SA}\frac{dP_{SA}}{dt} = Q_{Ao} - \frac{P_{SA}}{R_S} \qquad (10)$$

To ease implementation of this model in Simulink, we reorganize the expression of the ordinary differential equation as follows,

$$\frac{dP_{SA}}{dt} = \frac{1}{C_{SA}}\left(Q_{Ao} - \frac{P_{SA}}{R_S}\right) \ . \qquad (11)$$

The main window for our Simulink model is shown in Figure 3. The Integrator block corresponds to the pressure changes in the systemic artery. The model begins with a systemic arterial pressure of zero, and over time reaches a steady state condition. One of the most crucial components of our model is the pulsatile blood flow originating from the heart, modeled as Q_{AO}.

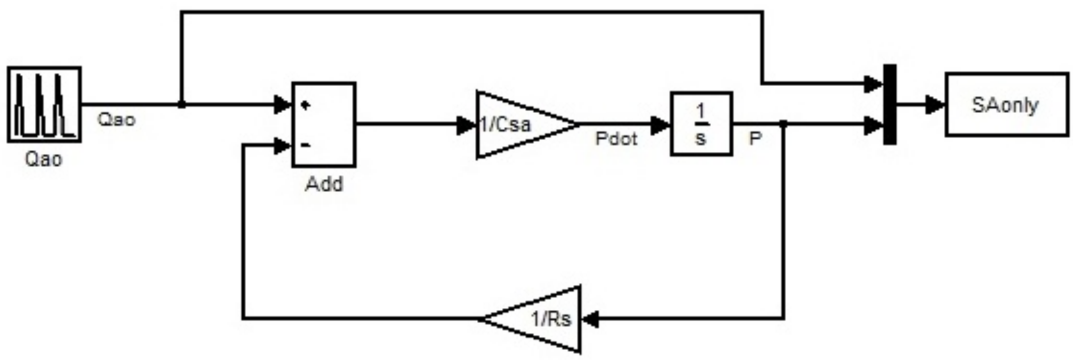

Figure 3: Main Window of Cardio_SA_only.mdl

Q_{AO} Waveform

The outflow from the heart into the systemic artery is denoted as $Q_{AO}(t)$. For the dynamic cardiac model that only includes the systemic artery, this outflow from the heart drives the dynamics of the model. We use the aortic flow that was modeled in Hoppensteadt and Peskin 2002, which simplifies the ejected volume as a triangle function during systole and zero during diastole. The parameters include Q_{max} (maximum blood flow), T_{max} (time when maximum blood flow occurs), T_S (duration of systole), T (duration of one heart beat). T_{max} corresponds with Q_{max}, which is the peak of our waveform. T_S corresponds with the length of the base of the triangle, and T is the period of the signal.

$$Q_{AO}(t) = \begin{cases} Q_{max}t \big/ T_{max} & 0 \le t \le T_{max} \\ Q_{max}(T_S - t) \big/ (T_S - T_{Max}) & T_{max} \le t \le T_S \\ 0 & T_S \le t \le T \end{cases} \qquad (12)$$

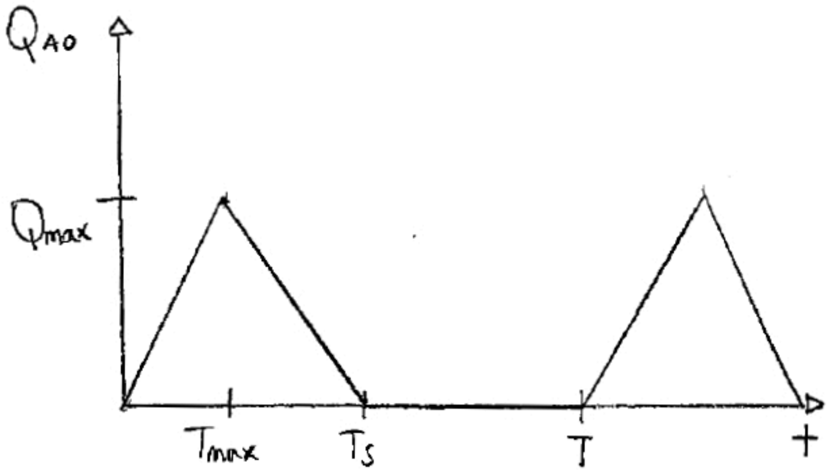

Figure 4: Outflow from heart into systemic artery denoted as $Q_{Ao}(t)$. Q_{max} is maximum blood flow occurring at T_{max}, T_S is duration of systole, and T is duration of one heartbeat. Time t measured in minutes

To model Q_{AO} in Simulink we used the Repeating Sequence block. The output values for the sequence were matched to their corresponding time values shown in Figure 5. A crucial component for the Repeating Sequence block to generate a reliable and consistent pressure output curve required that the control points of the cardiac output curve, Q_{Ao}, have to be a multiple of the simulation step size for the fixed step size simulation. It is also beneficial for the stop time for our simulation to be a multiple of the step size. We choose to model 16 heartbeats, and a step size of 75 ms. These values can be selected in the Configuration Parameters window, shown in Figure 6.

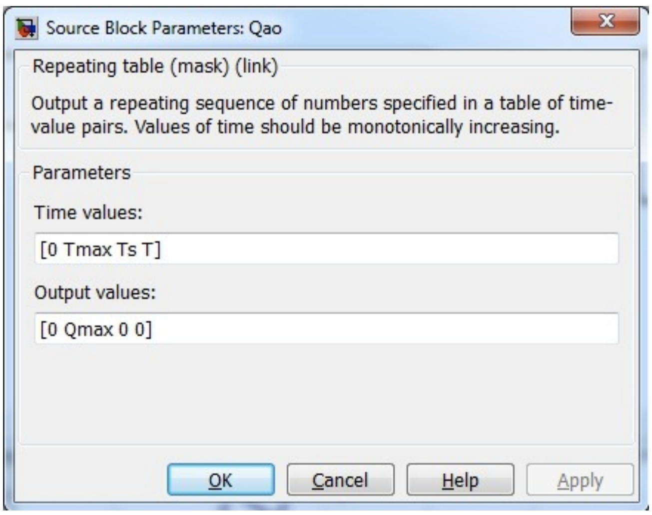

Figure 5: Repeating Sequence Parameters Window

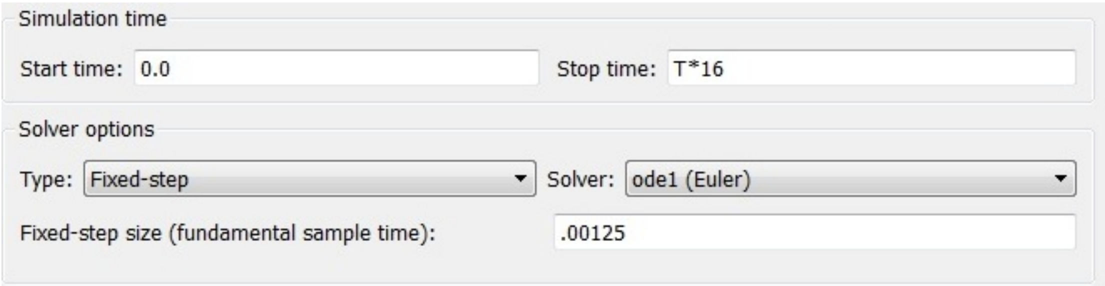

Figure 6: Configuration Parameters Window. The solver for our simulation is Fixed-Step with an ode1 (Euler) solver.

<u>*Running the Simulation*</u>

The Simulation will generate two plots with respect to time, one for Q_{AO} and the other for the blood pressure.

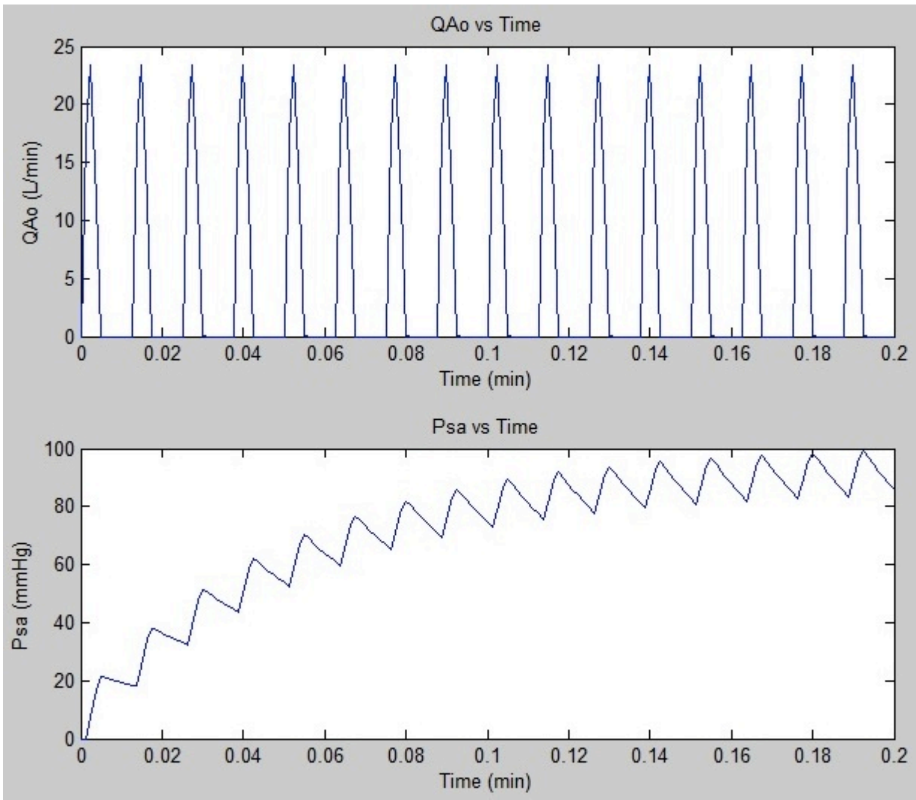

Figure 7: Scope of Cardio_SA_only.mdl. *They are both with respect to time (min) A) The top subplot is the output of the* Repeating Sequence *block for Q_{AO} B) The bottom subplot is the systemic arterial blood pressure.*

As mentioned earlier, the initial condition of the blood pressure was set to 0 mmHg. This "initial guess" of the arterial pressure is fairly far off of the actual value and several simulated heart beats must pass before the solution settles in to a stable solution. After multiple heartbeat cycles, the simulation starts to reach a steady state condition with a systolic pressure of approximately 100 mmHg and a diastolic pressure of 80 mmHg. Note that the reason it does not reach typical 120/80 mmHg pressure marker is due to our choice for the arterial compliance and systemic resistance. These values adjust the pulse pressure and mean arterial pulse of the arterial pressure curve.

The increases in pressure in Figure 7 correspond with the ejection of blood during systole. The decreases in pressure correspond with the duration of diastole. As the Q_{A0} signal produces more "heartbeats" the pressure curve begins to level out reaching a steady state condition.

Cardiovascular Model – Systemic Arterial and Left Ventricle

Dynamics of the Left Ventricle

As is, the first cardiovascular model does not address *how* the heart generates a pulsatile signal, the pulsatile ejected blood volume drives the dynamics of the model. However, interesting pathology can exist at the left ventricle that requires modeling of the contraction of the heart and the opening/closing of the valves. To accommodate this, we will expand the model to include the left ventricle LV. The left ventricle will be seen as a pump which "accepts fluids at low pressure (P_1) and transfers them into a region where the pressure is high ($P_2>P_1$)"[1]. The muscular contractions of this pump will change the compliance of the walls of the left ventricle, resulting in high compliance during diastole and low compliance during systole.

For this model, the input flowing blood is at low pressure and comes from the left atrium. During diastole, the left ventricle is relaxed and the pressure in the left ventricle is lower than the pressure in the systemic artery. This pressure differential keeps the outflow valve (aortic valve) closed. However, the small positive pressure differential between the left atrium and left ventricle keeps the inflow valve (mitral valve) open, allowing blood to flow into the left ventricle. This causes the left ventricle to expand due to its high compliance and the small increase in pressure. Once diastole is over and systole begins, the compliance of the left ventricle decreases as the muscles in the wall of the ventricle contract. This results in a large increase in pressure. With the pressure in the left ventricle exceeding that of the left atrium, the mitral valve closes. As the pressure in the left ventricle exceeds the arterial pressure, the aortic valve opens, and the blood is ejected as the left ventricle contracts. The ejected blood reaches pressure values close to 120 mmHg. In this sense, the heart is a pump which accepts blood at low pressures coming from the lung, and transfers it into the systemic arteries where pressures are higher.

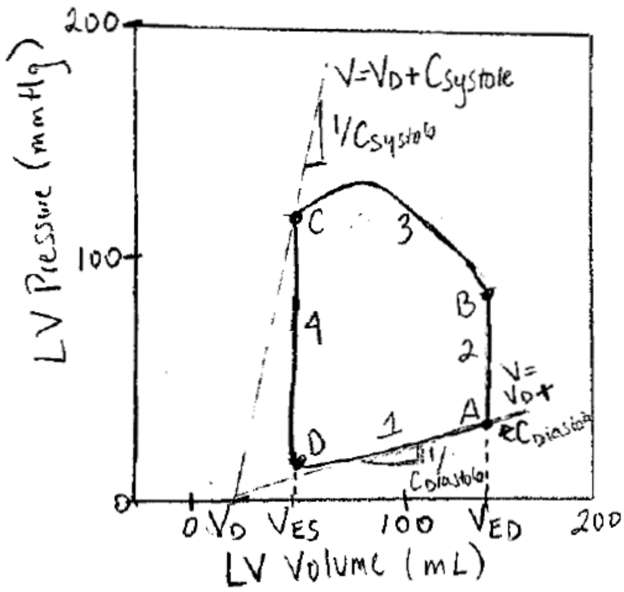

Figure 8: Pressure-Volume graph for left ventricle. On the volume axis we have labels for the "dead" volume remaining in the ventricle V_d, volume remaining at the end of systole V_{ES}, and volume remaining at the end of diastole V_{ED}. On the Pressure axis we have labels for the venous pressure P_v and arterial pressure P_a. The ventricular pressure volume loop follows one heartbeat. A: Mitral valve closes, B: Aortic valve opens, C: Aortic valve closes, D: Mitral valve closes. The two volume lines represent its relationship with pressure according to the compliance during systole and diastole, with the slope being 1/Compliance. On the vertical branches (AB and CD) the two valves are closed giving a constant volume. This is called isovolumetric contraction and isovolumentric relaxation respectively. On the horizontal branches (BC and DA) one valve is open allowing flow through. This is called ejection and filling respectively.

The left ventricle can be treated very similarly to the compliant vessels in the first cardiovascular model. However, compliance is no longer static. During diastole, the compliance increases accommodating larger volumes of blood. During systole, the compliance decreases, becoming rigid to contract ejecting blood with higher pressure. Therefore, we can modify our equations to include compliance as being time dependent. We use the time-dependence of compliance changes that are described in Hoppensteadt and Peskin, 2002. An example of this time-varying compliance is shown in Figure 9.

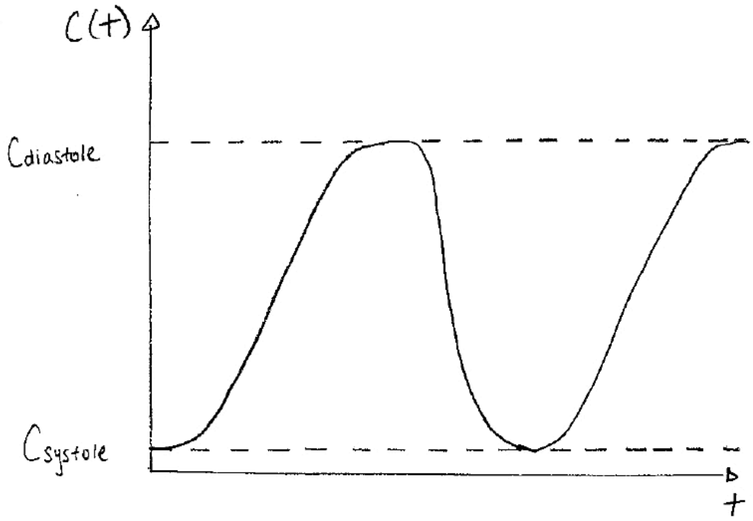

Figure 9: Compliance of ventricle as a function of time. During diastole compliance rises from $C_{Systole}$ to $C_{Diastole}$. During systole compliance lowers from $C_{Diastole}$ to $C_{Systole}$

We can begin our derivations for the left ventricle with Equation (4). Even though we have already derived equations for the systemic arteries, we will pair them up with the left ventricle equations.

$$V_{SA}(t) = V_{d,SA} + C_{SA}P_{SA}(t), \qquad (13)$$

$$V_{LV}(t) = V_{d,LV} + C_{LV}(t)P_{LV}(t), \qquad (14)$$

where $C_{LV}(t)$ is the time-varying compliance of the left-ventricle, $P_{LV}(t)$ is the time-varying pressure in the left-ventricle, and $V_{LV}(t)$ is the time-varying volume of blood in the left ventricle. To find the change in volume over time, we take the derivative of both equations.

$$\frac{dV_{SA}}{dt} = C_{SA}\frac{dP_{SA}}{dt} \qquad (15)$$

To take the derivative of Equation (14), we must use the product rule because both compliance and pressure are dependent on time.

$$\frac{dV_{LV}}{dt} = C_{LV}(t)\frac{dP_{LV}}{dt} + P_{LV}(t)\frac{dC_{LV}}{dt} \qquad (16)$$

Finally, we can also express changes in volume as the difference between blood flows at the input and output of the model segments.

$$\frac{dV_{SA}}{dt} = Q_{Ao} - Q_S \qquad (17)$$

$$\frac{dV_{LV}}{dt} = Q_{Mi} - Q_{Ao} \qquad (18)$$

where $Q_{Mi}(t)$ is the flow through the mitral valve and $Q_{Ao}(t)$ is the flow through the aortic valve. Since we will be integrating over differential pressures in our model and not volumes, let us reorganize Equations (15) and (16) to the first order differentials equations involving pressures. This will ease the implementation of our model in Simulink

$$\frac{dP_{SA}}{dt} = \frac{1}{C_{SA}}\left(\frac{dV_{SA}}{dt}\right) \qquad (19)$$

$$\frac{dP_{LV}}{dt} = \frac{1}{C_{LV}(t)}\left(\frac{dV_{LV}}{dt} - P_{LV}\frac{dC_{LV}}{dt}\right) \qquad (20)$$

We initialize the systemic arterial pressure at time = 0s to 80 mmHg, and the left ventricular pressure at time = 0s to 5 mmHg, the amount of pressure in the left atrium determined by the venous return from the pulmonary circulation.

Blood Flow through Mitral and Aortic Valves

Blood flow through the mitral and aortic valves can be adapted from Equation (1).

$$Q_{Mi} = S_{Mi}\frac{(P_{LA} - P_{LV}(t))}{R_{Mi}} \qquad (21)$$

$$Q_{Ao} = S_{Ao}\frac{\left(P_{LV}(t) - P_{SA}(t)\right)}{R_{Ao}} \qquad (22)$$

where S_{Mi} and S_{Ao} are indicator variables that reflect if a valve is open or closed. If a valve is open, the "S" variable will have a value of 1, and if closed, it will have a value of 0. An open valve allows forward flow through it, while a close valve prevents backflow from occurring. To accomplish this, S_{Mi} and S_{Ao} use the sign of the pressure difference to determine the direction of flow, and hence, if a valve should be open or closed. A positive pressure difference equals forward flow, while a negative pressure difference equals backflow. By allowing other values besides simply 1's or 0's, the model can accommodate leaky valves or valve insufficiency.

Hoppensteadt and Peskin assume valves have some resistance to them, and a pressure drop exists across them due to the blood flowing through the valve. The resistance is small and hence the pressure drop is also small. Healthy values have low resistances, having negligible impacts on circulation.

S_{mi} is:
0: $P_{LA} < P_{LV}$
1: $P_{LA} > P_{LV}$
(23)
S_{Ao} is:
0: $P_{LV} < P_{SA}$

$$1: P_{LV} > P_{SA}$$

In Simulink we use the Relational Operator block with a > sign chosen to compare the two pressure values. This generates a logical output of 1 or 0 which is turned into a double value using the Data Type Conversion block.

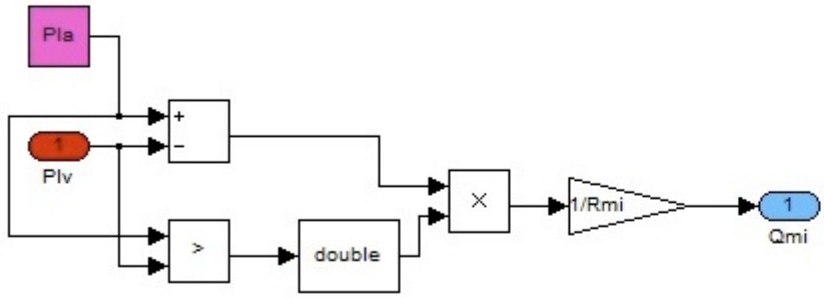

Figure 10: Q_{MI} subsystem. P_{LV} is an input into the subsystem, and Q_{Mi} is an output. This subsystem follows Equation (21)

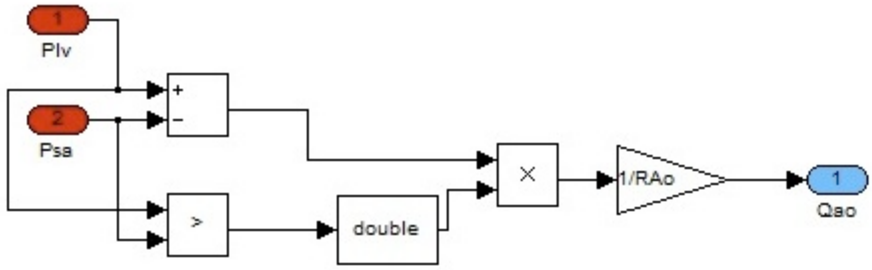

Figure 11: Q_{Ao} subsystem. P_{LV} and P_{SA} are inputs into the subsystem, and Q_{Ao} is an output. This subsystem follows Equation (22)

Left Ventricular Compliance Changes Over Time

One of the driving functions in our model is the left ventricular compliance. The periodic function we use follows the qualitative sketch from Figure 9. It is defined below for one period, from Hoppensteadt and Peskin, 2002:

$$C_{LV}(t) = \begin{cases} C_{LVD} * \left(\dfrac{C_{LVS}}{C_{LVD}}\right)^{\frac{1-e^{-t/\tau_S}}{1-e^{-T_S/\tau_S}}}, & 0 \le t \le T_S \\ C_{LVS} * \left(\dfrac{C_{LVD}}{C_{LVS}}\right)^{\frac{1-e^{-(t-\tau_S)/\tau_D}}{1-e^{-(T-T_S)/\tau_D}}}, & T_S \le t \le T \end{cases} \tag{24}$$

It is a piecewise function, divided into two sections, for systole and diastole. The function transitions between the low compliance C_{LVS} found during systole, and the high compliance C_{LVD} found during diastole. The transitions are determined by the time constants τ_S and τ_D.

Equation (24) is found in the Clv(t) subsystem within the LV Compliance subsystem which uses a Switch block to alternate between the piecewise functions depending on the time variable relative to the heart cycle.

The Switch block will output either its first or third input as the left ventricular compliance depending on the internal condition set relative to its second input. The first input is the output of the piecewise function related to diastolic compliance, and the third input is the output of the piecewise function related to systolic compliance. The second input must be greater than the threshold of T_S for the first input to be selected. Otherwise the third input will be used.

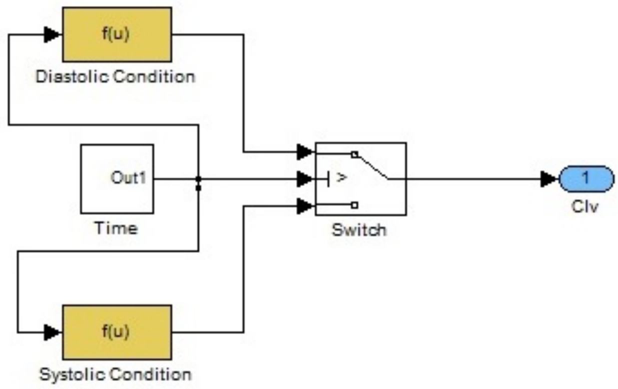

Figure 12: Clv(t) Subsystem. The Diastolic Condition FCN block holds the piecewise function in Equation (24) related to diastole and is the Switch block's first input. The Systolic Condition FCN block holds the piecewise function in Equation (24) related to systole and is the Switch block's third input. The Time subsystem calculates the time input to be used for Diastolic and Systolic Condition, as well as the Switch's second input.

To determine which of the two functions to use, the Clv(t) subsystem manipulates the time of simulation to reset itself to zero at the end of diastole, effectively dealing with one period of the signal. To accomplish this, the Time subsystem uses the remainder (rem) function to find the remainder of the current time of the simulation (obtained from the Clock source block) divided by the period T.

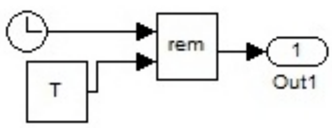

Figure 13: Time Subsystem.

Running the Simulation

With the necessary derivations complete for all the elements of the model, let us look at how it is built on Simulink. The model uses several sets of equations to observe the changes in $P_{LV}(t)$, $P_{SA}(t)$, $V_{LV}(t)$, $V_{SA}(t)$, $Q_{Mi}(t)$, $Q_{Ao}(t)$, $Q_S(t)$, and $C_{LV}(t)$. The pairs of equations used to calculate $P_{LV}(t)$, $P_{SA}(t)$, $V_{LV}(t)$, and $V_{SA}(t)$ are Equations (13,14 and 19,20). The pairs of equations used to calculate $Q_{Mi}(t)$, $Q_{Ao}(t)$, and $Q_S(t)$ are Equations (9,21,22). The equation used to calculate $C_{LV}(t)$ is Equation (24).

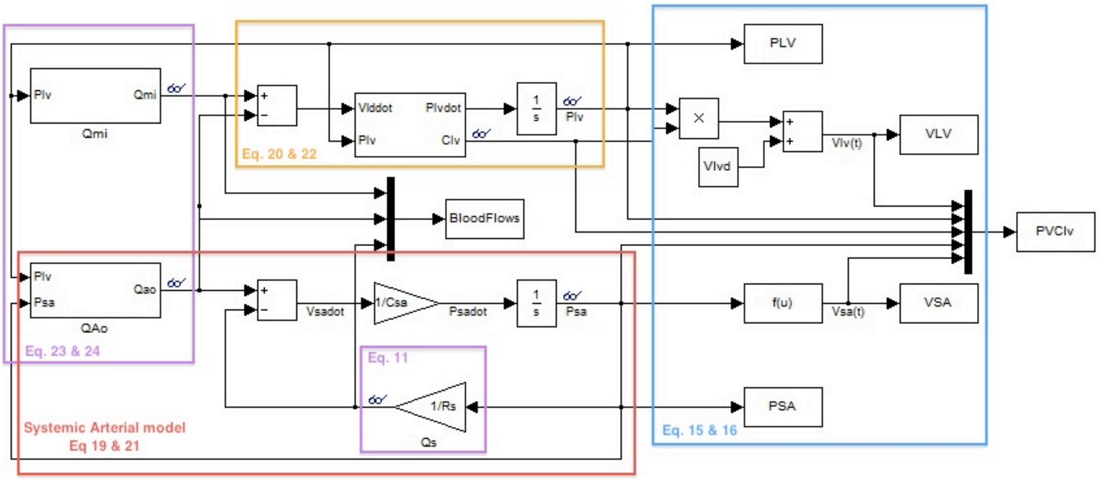

Figure 14: Main Cardio_SA_LV Simulink Window. Bottom segment is systemic arterial model, top segment is addition of left ventricular model.

Notice that $P_{LV}(t)$, $P_{SA}(t)$, $V_{LV}(t)$, and $V_{SA}(t)$ are exported to the MATLAB workspace as cell structures through the To Workspace blocks. Because Simulink only plots with respect to time, they need to be exported in order to generate plots for the pressure-volume loops of left ventricle and systemic arteries.

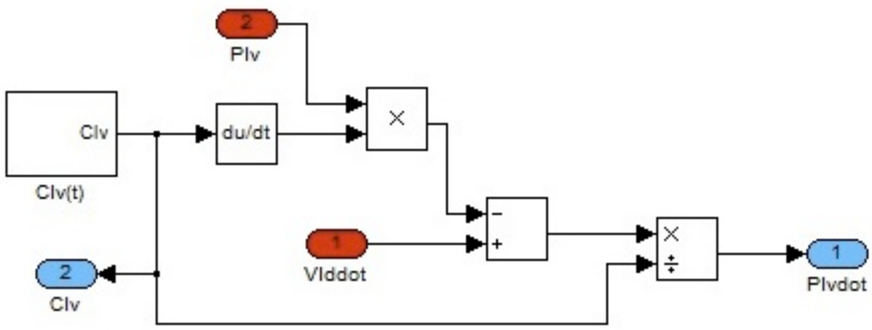

Figure 15: LV Compliance subsystem. Uses Equation (20) and holds Clv(t) subsystem.

The results of the simulation are shown in Figure 16, 17, and 18. From the top subplot in Figure 16 we have the left ventricular compliance. The plot our simulation generates is similar to the qualitative sketch from Figure 9. We see that during diastole the compliance increases to its maximum of C_{LVD} and during systole it decreases to its minimum of C_{LVS}. The transitions between the two match well with the increases and decreases in blood pressure and flow from P_{LV} and Q_{Ao}.

In the middle subplot we can see the plotted pressure waves. The blue left ventricular pressure graph jumps between 110 mmHg and 5 mmHg reaching the expected values of pressure during contraction and filling. The green systemic arterial pressure holds the steady state condition of 110 mmHg / 80 mmHg we had in our previous model. Because we initialized it to 80 mmHg instead of 0 mmHg, it did not require much time for it to reach equilibrium.

The bottom subplot shows the blood flows through the aorta, mitral valve and systemic arteries. We see that during systole, the blood flow through the aorta spikes, reaching a maximum of 84 L/min as the heart is contracting ejecting blood. During the same time frame, we see no blood flow through the mitral valve, which we expect since it is closed. We can clearly see when the aortic valve closes and the mitral valve opens, as the aortic blood flow falls to zero and the mitral blood flow jumps to 25 L/min. The difference in flows between the two valves can be related to their pressure. Remember, we expect the pressure drop across the aortic valve to reach levels of 120-80 = 40 mmHg, while the pressure drop across the mitral valve is close to 5 mmHg. The higher pressure drop means more blood flow through it. In the systemic arteries we have a near constant flow of 5 L/min slightly increasing during systole, and decreasing during diastole.

Figure 17 illustrates the ventricular pressure volume loop sketched in Figure 8. The output we get in MATLAB matches well with the sketch. Isovolumetric contraction and relaxation are both vertical straight lines, and in ejection we see the increasing pressure and curve right before the aortic valve closes. During filling we see a slight increase as the left ventricle fills up with blood.

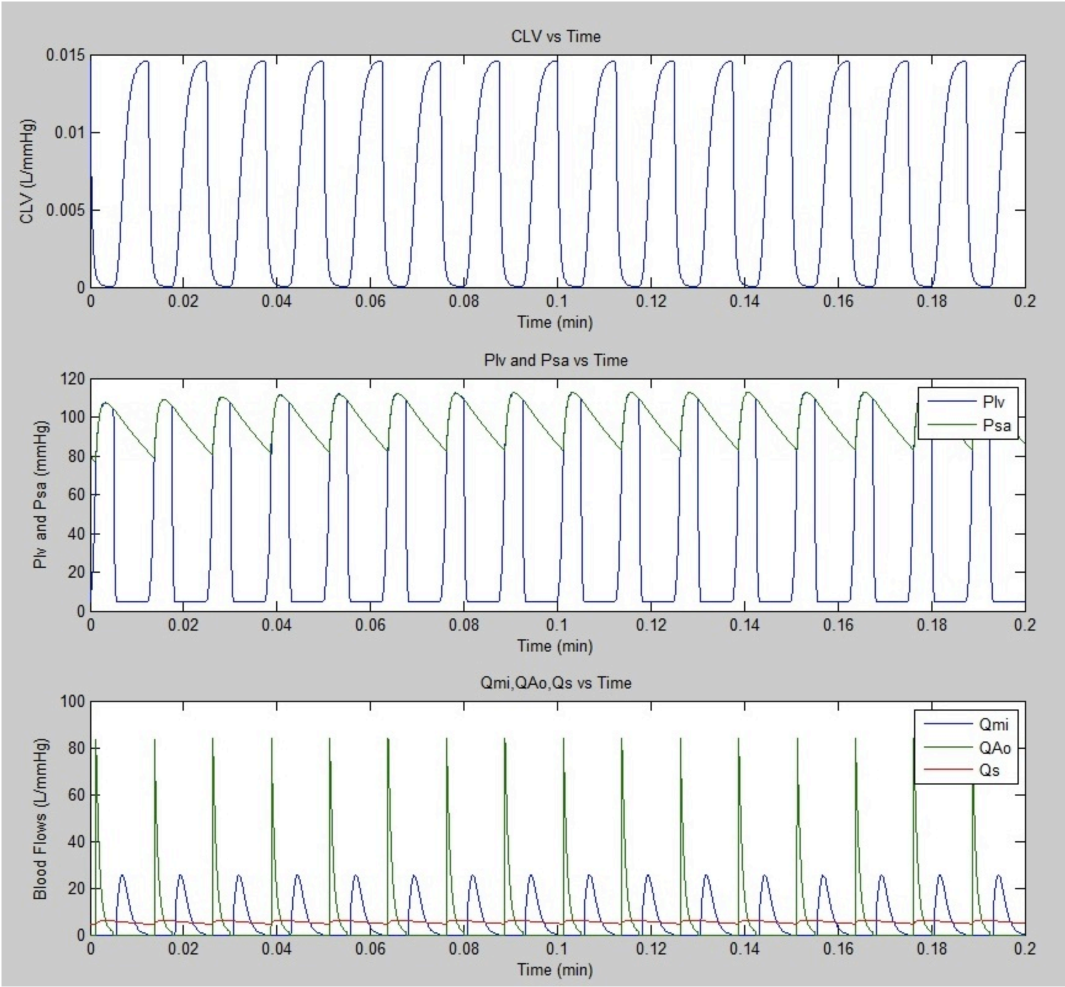

Figure 16: Plot of results: Top) LV Compliance as a function of time Middle) $P_{LV}(t)$ and $P_{SA}(t)$ plotted as functions of time. Bottom) Blood Flows as functions of time.

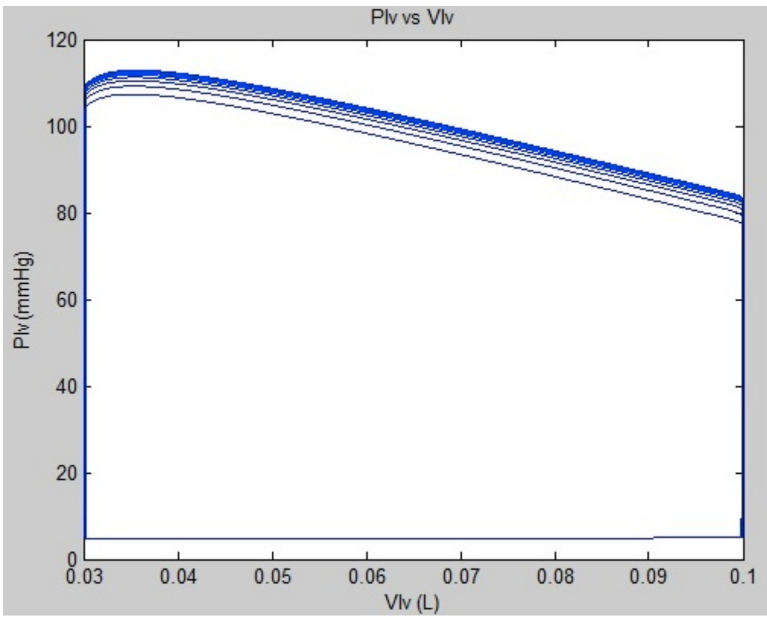

Figure 17: Left Ventricular Pressure-Volume Loop

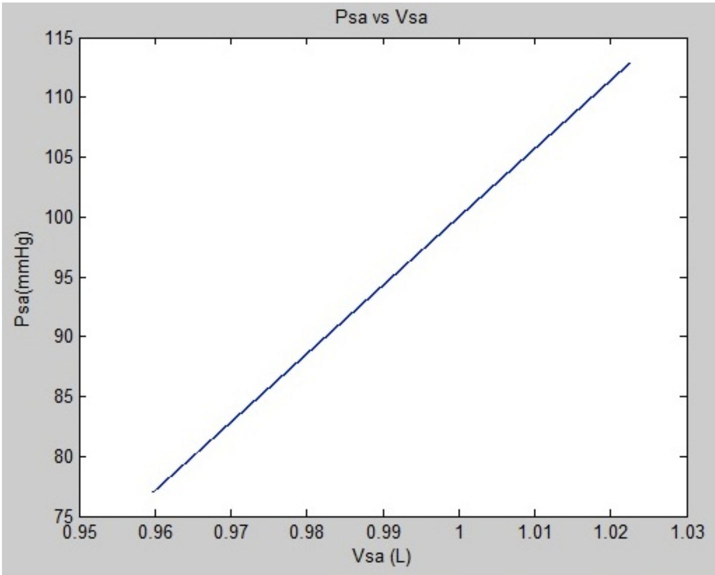

Figure 18: Systemic arterial pressure vs. Systemic arterial volume

Model Applications

We can adjust our model to reflect diseases or conditions in the body. For example, if we were to model mitral and aortic stenosis, we can change the values of R_{Mi} and R_{Ao} accordingly.

Mitral Stenosis

To model mitral stenosis let us begin by changing the value of R_{mi} by a magnitude of 10 in the parameters_SA_LV.m file.

At a first glance, from Figure 19, the model doesn't seem to change its behavior. P_{LV} seems to reach the same 110 mmHg mark, and P_{SA} is still in the range of 110 mmHg / 80 mmHg. However, taking a closer inspection, at the bottom of the graph P_{LV} drops below the consistent 5 mmHg to 3 mmHg, and then goes back up to 5 mmHg. This is to be expected, mitral stenosis would only affect during diastole, when the valve is open, with a greater resistance value we should expect a greater pressure drop through it.

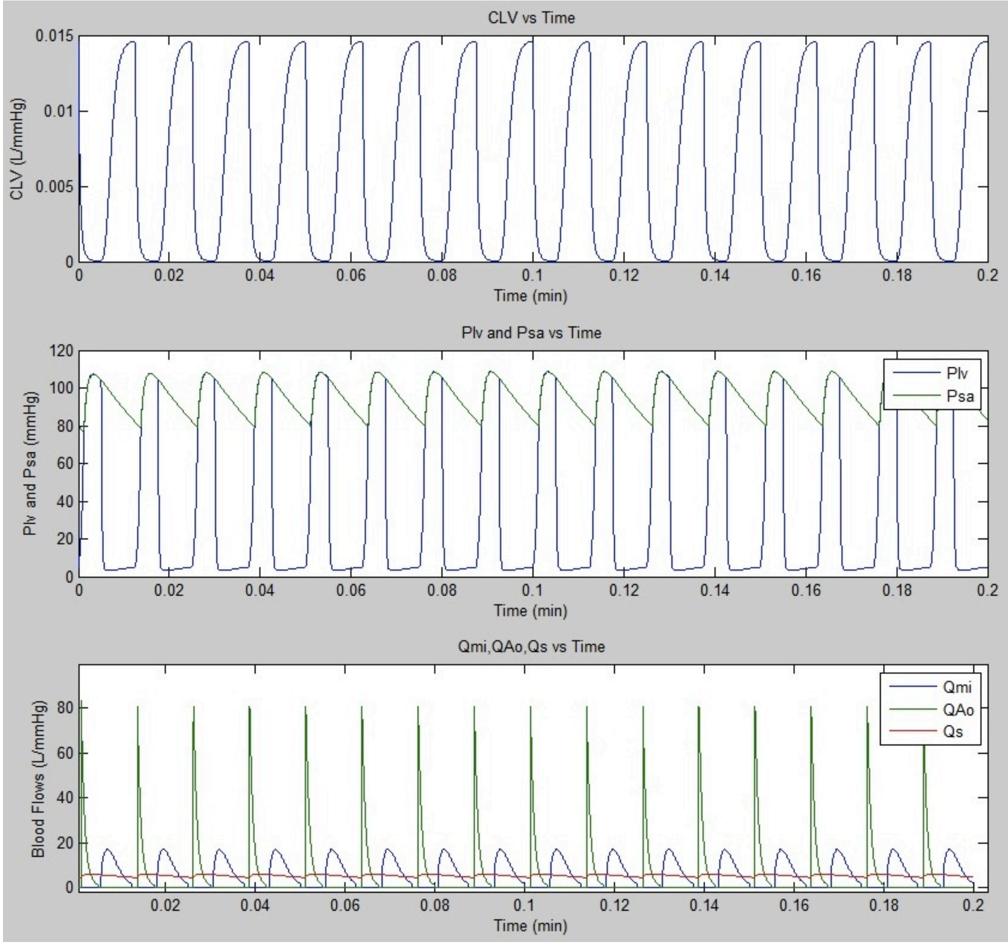

Figure 19: Mitral Stenosis

But perhaps the biggest change we can see between the simulations, is that Q_{Mi} has a flow difference of 8 L/min less than in the previous simulation. The stenosis doesn't allow adequate blood flow through it, and this is demonstrated by the decrease in Q_{Mi}.

Aortic Stenosis

If we instead take a look at aortic stenosis, we get a much different result. By changing the aortic resistance to 1 mmHg/(L/min), the left ventricular pressure jumps by almost 40 mmHg to 140 mmHg! The increased resistance prevents flow through the valve, and the pressure inside the ventricle builds up to extremely high levels.

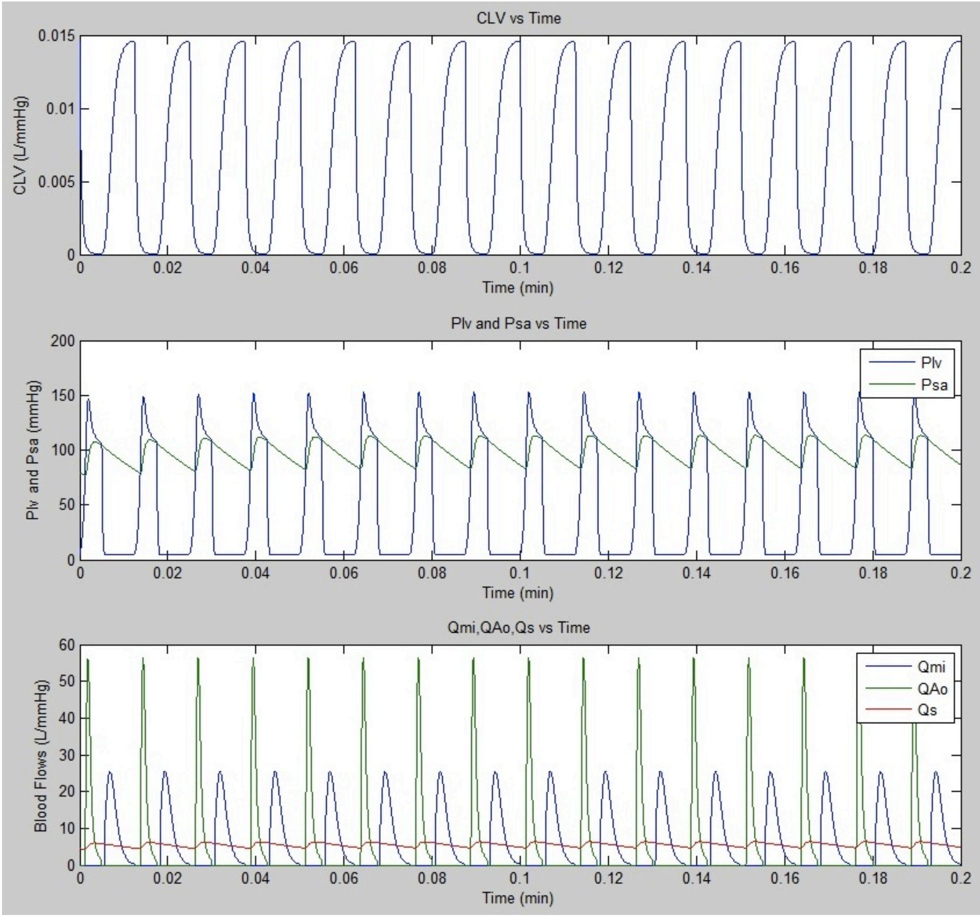

Figure 20: P_{LV} and P_{SA} with Aortic Stenosis

Consequently, this will also affect the left ventricular pressure volume loop. In Figure 21, we see that as soon as the aortic valve opens the pressure balloons up indicated by the value of Plv in the middle subplot on Figure 20. The high resistance in the aortic valve allows the buildup of pressure in the left ventricle due to the compliance changes in the heart. As blood is ejected from the left ventricle, the pressure falls to a more normal value.

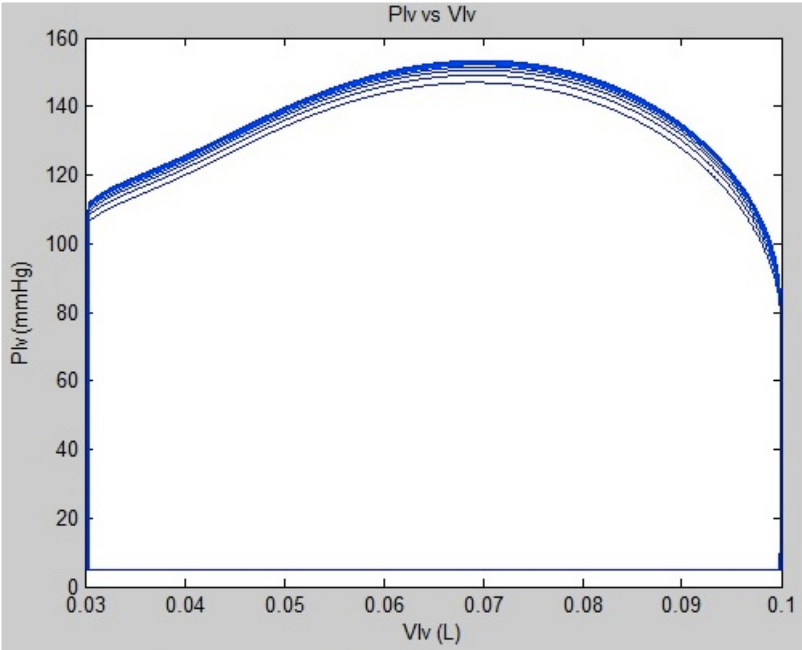

Figure 21: Ventricular Pressure Volume Loop with Aortic Stenosis

Works Cited

[1] Hoppensteadt, Frank C., Peskin, Charles S. Modeling and Simulation in Medicine and the Life Sciences. 2nd Ed. New York: Springer Science+Business Media, Inc, 2002. Print

Appendix 1 – Parameter Files

Parameter File for Cardio_SA_only.mdl

```
%This Model has been adapted from the MATLAB code presented in
%Hoppensteadt and Peskin "Modling and Simulation in Medicine and the
%Life Sciences

%Time parameters
T = 0.0125;                      %Duration of heartbeat: min
Ts = 0.0050;                     %Duration of systole: min
Stroke_volume = 70e-3;           %Volume ejected by 1 heart beat: L
Tmax = 0.0020;                   %Time at which flow is max: min
Qmax= Stroke_volume/(0.5*Ts);    %Max flow through aortic valve: L/min
dt = .01*T;      %This choice implies 100 timesteps per cardiac cycle

%Compliance and resistance values for arteries
Csa = .003;                    %Systemic arterial compliance: L/mmHg
Rs = 17.86;                    %Systemic resistance: mmHg/(L/min)
```

Parameter File for Cardio_SA_LV.mdl

```
%This Model has been adapted from the MATLAB code presented in
Hoppensteadt
%and Peskin "Modling and Simulation in Medicine and the Life Sciences

%Time parameters
T = 0.0125;                      %Duration of heartbeat: min
Ts = 0.0050;                     %Duration of systole: min
Stroke_volume = 70e-3;           %Volume ejected by 1 heart beat: L
Tmax = 0.0020;                   %Time at which flow is max: min
Qmax= Stroke_volume/(0.5*Ts);    %Max flow through aortic valve: L/min
dt = .00005*T;                   %This choice implies 20,000 timesteps
per cardiac cycle

%Compliance and resistance parameters. Note that valve resistances
are not
%supposed to be realistic, just small enough to be negligible
Csa = .00175;                   %Systemic arterial compliance: L/mmHg
Rs = 17.86;                     %Systemic resistance: mmHg/(L/min)
Rmi = .01;                      %mitral valve resistance: mmHg/(L/min)
RAo = .01;                      %Aortic valve resistance: mmHg/(L/min)

Vlvd = .027;                    %Left ventricular volume when PLV=0: L
Vsad = .825;

%Systemic arterial volume when Psa+0: L
Pla = 5;                        %Left atrial pressure: mmHg

%Parameters for Clv(t)
CLVD = .0146;

%Max (diastolic) value of CLV: L/mmHg
CLVS = .00003;

%Min (systolic) value of CLV: L/mmHg
tauS = .0025;                  %CLV time constant during systole: min
tauD = .001;                   %CLV time constant during diastole: min

%Initialization parameters
Plvi = 5;                        %Initial value of Plv: mmHg
Psai = 80;                       %initial value of Psa: mmHg
```

Appendix 2 – MATLAB Functions

Plotting Scripts

```
%This script will plot the QAo and Psa from the SA only cardio model.
%Store the arrays for future plotting:
time=SAonly.time;
QAo=SAonly.signals.values(:,1);
Psa=SAonly.signals.values(:,2);

%Plot Results
figure(1)
subplot(2,1,1), plot(time,QAo);
title('QAo vs Time');
ylabel('QAo (L/min)');
xlabel('Time (min)');
subplot(2,1,2), plot(time,Psa);
title('Psa vs Time');
ylabel('Psa (mmHg)');
xlabel('Time (min)');

%This script will plot the three blood flows, the pressure and volume
%of the LV and SA, the pressure-volume loops and the compliance
%changes

%Store the arrays for future plotting:
time=BloodFlows.time;
Qmi=BloodFlows.signals.values(:,1);
QAo=BloodFlows.signals.values(:,2);
Qs=BloodFlows.signals.values(:,3);
Vlv=PVClv.signals.values(:,1);
Plv=PVClv.signals.values(:,2);
Clv=PVClv.signals.values(:,3);
Psa=PVClv.signals.values(:,4);
Vsa=PVClv.signals.values(:,5);

%Plot Results
figure(1)
subplot(3,1,1), plot(time,Clv);
title('CLV vs Time');
ylabel('CLV (L/mmHg)');
xlabel('Time (min)');
subplot(3,1,2), plot(time,Plv,time,Psa);
title('Plv and Psa vs Time');
ylabel('Plv and Psa (mmHg)');
xlabel('Time (min)');
legend('Plv','Psa');
subplot(3,1,3), plot(time,Qmi,time,QAo,time,Qs);
title('Qmi,QAo,Qs vs Time');
ylabel('Blood Flows (L/mmHg)');
xlabel('Time (min)');
legend('Qmi','QAo','Qs');

%LV pressure-volume loop
figure(2)
plot(Vlv,Plv);
title('Plv vs Vlv');
ylabel('Plv(mmHg)');
xlabel('Vlv (L)');

%SA pressure-volume loop
figure(3)
```

```
plot(Vsa,Psa);
title('Psa vs Vsa');
ylabel('Psa(mmHg)');
xlabel('Vsa (L)')
```

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