A Stylized History of Quantitative Finance

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Modern Finance in One Sentence

Feynman: *Physics/Medicine* in one sentence.

Finance: For Any Security:

IF

you can hedge away all correlated risk

AND

you can then diversify over all uncorrelated risk

THEN

you should expect only to earn the riskless rate

What does this assume?

- Stable frequentist statistics of diffusion
- Replication (of a riskless bond)
- The Law of One Price

The history …

Summary

- Derivatives *sans* Diffusion
- Diffusion *sans* Derivatives
- 1952: Markowitz: Risk as Volatility
- 1958: The Idea of Replication and The Law of One Price
- If you can eliminate all risk, you must have replicated a riskless bond whose return you know: CAPM
- Why is CAPM bad?
- 1960s: Options Models: Derivatives + Diffusion + Volatility + Hedging + Replication
- Why is Black-Scholes-Merton better than CAPM?
- Business Time vs Calendar Time
- 1976: Calibration: The Invention of Implied Volatility
- Volatility as an Asset Class
- 1977: Vasicek on modeling parameters rather than assets
- The Smile
- Even More Calibration
- The Future: What makes a market?
- Behavioral Finance
- Market Microstructure

The Great Idea of Derivatives

- This certainly goes back thousands of years, but … jumping to 17th Century: Spinoza
- Spinoza's treats emotions like Euclid treats geometry: emotions are derivatives.
  - Primitives are Desire, Pleasure, Pain (cf. Equity, Fixed Income, Credit)
- *Good* is everything that brings pleasure, and *Evil* is everything that brings pain.
- Love: Pleasure associated with an external object.
- Hate: Pain associated with an external object.
- Envy: Pain at another's Pleasure.
- Schadenfreude
- Cruelty: Desire to inflict Pain on a someone Loved.
- Three more primitives:
  - Vacillation, Wonder, Contempt.

Pleasure Pain Desire: A Map of the Emotions*

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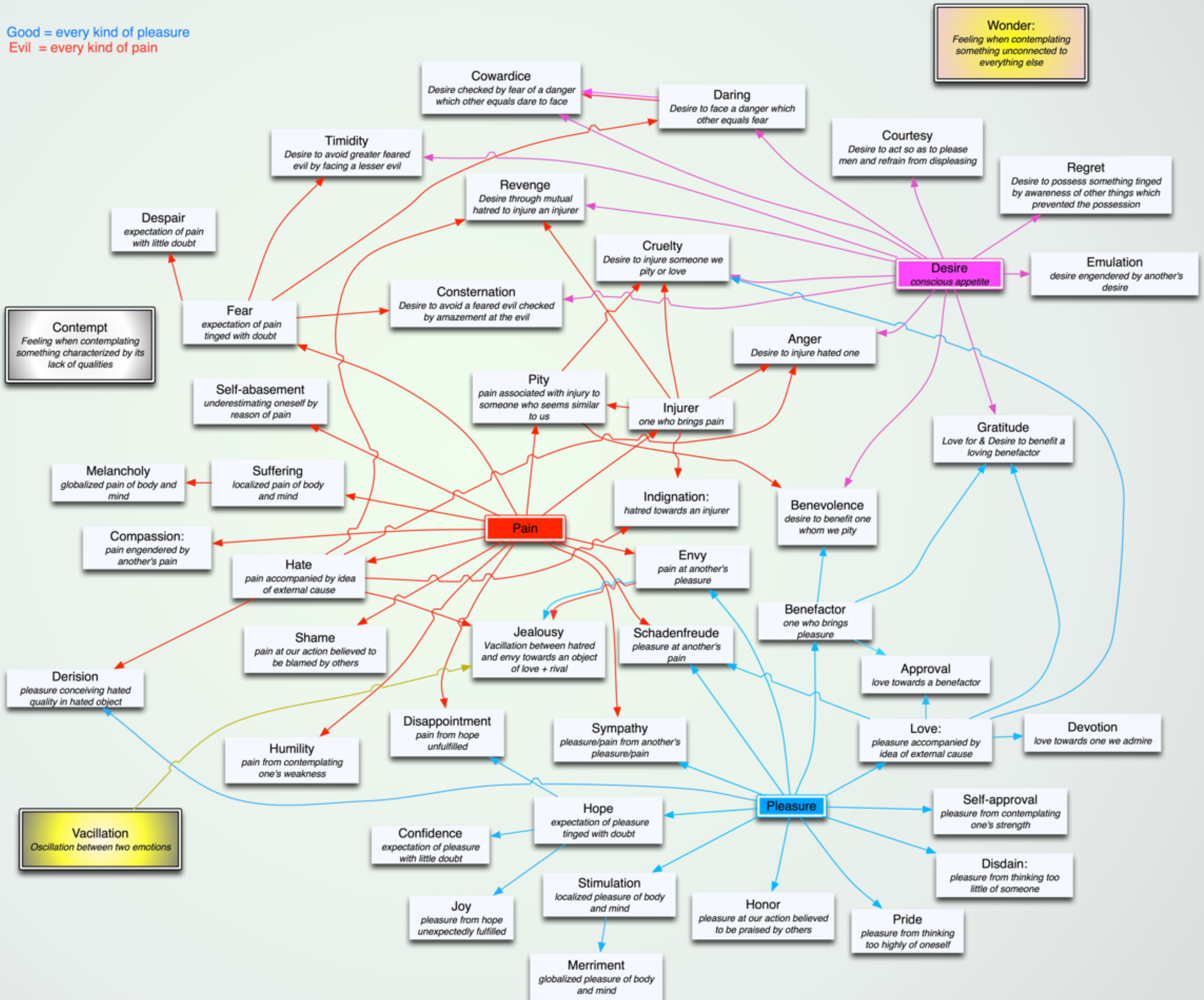


according to *Ethics*, Benedict de Spinoza

Derivatives *sans* Diffusion

- Spinoza is very modern: the passions gain their value from their relationship to underlying sensations.
- Spinoza believes that human behavior behaves laws, that nothing is random.
- But his scheme and definitions are static: there is almost no possibility of motion, except for

Vacillation: the cyclic alternation between two different passions.

Jealousy is the oscillation between Hatred and Envy in relation to a Love object and a rival

Hatred is Pain associated with an external Person

Envy is Pain at another's Pleasure.

Vacillation involves volatility – the more rapidly and intensely one Vacillates, the greater the Jealousy.

- Spinoza has no Anxiety in his system. Various opinions:

  Anxiety is a vacillation between Hope and Fear;

  Anxiety is not a Passion;

  There was no Anxiety in the 17th Century.

Diffusion *sans* Derivatives

- 1831: Thomas Graham
  "…gases… when brought into contact, do not arrange themselves according to their density, … but they spontaneously diffuse, mutually and equally, through each other, and so remain in the intimate state of mixture for any length of time."

- 1858: James Clerk Maxwell
  the first theory of atoms moving in gases

- Ludwig Boltzmann,
  Boltzmann equation: kinetic theory derives the properties of matter from the properties of atoms

- Early 20th Century.
  Albert Einstein, Marian Smoluchowski and Jean-Baptiste Perrin confirm atomic theory of matter.

- Physicists understood diffusion not derivatives.They discussed the behavior of underliers, but not functions of underliers.

- Except: Bachelier in the opening year of the 20th Century developed and applied diffusion to finance and derived the equations for Brownian motion applied to options with underliers that undergo arithmetic Brownian motion. (Ahead of his time, rediscovered in the 1960s)

1952:Risk = Volatility

- Harry Markowitz measures “expected return” against “risk”.

  Risk = the standard deviation of returns.

  Suggests finding the portfolio with the most return for a given risk

1958: The Idea of Replication and The Law of One Price

- Modigliani and Miller: You can recreate (and value) a levered firm for yourself by borrowing money to buy the stock of an unleveraged firm.

- Replication as a strategy for valuation

Early 1960s: CAPM/APT

- Finance's Key Question: What return should you expect from taking on risk?
- Use Markowitz definition of risk as standard deviation
- **Replication and the Law of One Price leads to CAPM and APT as well as BSM.**
- The only currently **known** return is *r*, that of a riskless bond.
  The valuation strategy is to link the **unknown** return on any security to *r* by reducing the security's risk to zero by including it in a portfolio.

How Can You Reduce Risk?

- Dilution:
  Combine security with a riskless bond
- Diversification:
  Combine security with many uncorrelated securities
- Hedging:
  Combine security with a correlated security
- Apply this in three successively more realistic worlds

Simple World 1
A few uncorrelated stocks and a riskless bond:
More risk, more return
All stocks have same Sharpe Ratio

- Dilution: Combine weight w of a risky stock S (μ,σ) with a weight (1 - w) of a riskless bond B (r,0) to create a new security with lower risk & return

$$[w\mu + (1-w)r, w\sigma] = [r + w(\mu - r), w\sigma]$$

- Law of One Price:
All uncorrelated stocks with risk $w\sigma$ earn excess return $w(\mu - r)$

- One parameter fixes everything

$$\frac{\mu - r}{\sigma} = \lambda$$

- Same Sharpe ratio for all stocks!

Less Simple World 2:

Many Uncorrelated Stocks: Diversify!

Every stock is expected to earn the riskless rate *r*

The Sharpe Ratio is zero

- Suppose there are countless uncorrelated stocks (μ_i, σ_i)
- Put them all in a portfolio with weights: $P = \sum w_i S_i$
- Then the portfolio risk σ diversifies to zero.

$$\mu - r = \lambda\sigma = \lambda\sum w_i\sigma_i = 0$$

- Thus the portfolio is riskless: $\mu = r$
- But the portfolio return is the sum of individual returns:

$$\mu = \sum w_i\mu_i = \sum w_i(r + \lambda\sigma_i) = r + \lambda\sum\sigma_i \quad \text{therefore} \quad \lambda = 0$$

- Thus every stock is expected to earn the riskless rate!

More Realistic World 3: CAPM

All Stocks are Correlated with the Market: Hedge Market Risk, Then Diversify! Sharpe Ratio of Every Market-Neutral Stock is Zero.

Every market-neutral stock must earn the riskless rate.

- Suppose there are countless stocks S_i correlated with the market M
- Then the market-neutral stock $S_i^M = S_i - \beta_i\left(\left|\frac{S_i}{M}\right|\right)M$ is uncorrelated with M
- After diversification each market-neutral stock S_i^M earns riskless rate.
- Which means $(\mu_i - r) = \beta_i(\mu_M - r)$
- CAPM just says that if you hedge every stock with the market, and then diversify over all remaining risk, you should earn only the riskless rate.

Why is CAPM Bad?

- Because Risk is Not Really the Standard Deviation of Returns
- And the market M and the stock S are not really stably correlated.
- Markets are not exactly like flipping coins. There isn't a well-defined frequentist probability of a market crash.

1960s:
Early Options Models:
Diffusion and Volatility but No Replication

- Samuelson, Sprenkel, Ayres, Boness … value stock options actuarially, as the expected discounted payoff of the option under a lognormal distribution with a growth rate and a volatility.

- But at what rate does the stock grow?

- And what discount rate to use?

- If you demand consistency with put-call parity, you can guess that the right rate is the riskless rate. Why did no one guess that?

1973: Putting everything together
Black-Scholes-Merton
Diffusion+Volatility+Hedging+Replication

- Use diffusion for the move in the underlying stock price dS
- Use stochastic calculus to find the move in the derivative $dC(S)$
- Hedge to eliminate stock risk from option: $dC - \Delta\, dS$
- Require that hedged portfolio, which is riskless, earns the known riskless rate r:

$$dC - \Delta\, dS = r(C - \Delta\, dS)$$

- Then we get the same formula as the actuarial one, but where all growth and discount rates are replaced by the riskless rate r.

Black-Scholes-Merton

- You can replicate/hedge an option with stock
- Option C and stock S must have same Sharpe ratio

$$\frac{\mu_S - r}{\sigma_S} = \frac{\mu_C - r}{\sigma_C}$$

- Ito's Lemma applied to a C leads to Black-Scholes

$$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 = rC \qquad \text{Black-Scholes Equation}$$

- A unified treatment of BSM and CAPM from one principle

Why is BSM better than CAPM?

- Because you really can hedge an option with a stock, because the correlation is really 1.
- So even if you don't believe the risk is the standard deviation of returns, the two securities really are truly connected, unlike the statistical connection between two different stocks.
- We have assumed that volatility is unchanging! If volatility is random, then the derivative is not really a derivative except at expiration.

1970s: Using the BS Equation

- Now, instead of forecasting the return of the stock, traders must forecast the volatility of the stock.

$$\frac{\partial C}{\partial t} + rS\frac{\partial C}{\partial S} + \frac{1}{2}\frac{\partial^2 C}{\partial S^2}\sigma^2 S^2 = rC \qquad \text{Black-Scholes Equation}$$

- Black and Scholes set about using the equation by using historical volatilities to estimate future volatilities. But who knows what future volatility will be?

1973: Use Business Time for Measuring Risk and Return

- P. C. Clark: The idea of examining market movements with a clock that doesn't tick seconds, but ticks number of trades or volume of trades. A behavioral approach to time perception.
- Returns seem to be normally distributed when examined as return per tick rather than return per second. (Geman, Ané)
- If all stocks have the same Sharpe Ratio **in business time**, then, in calendar time:

$$\frac{\mu - r}{\sigma\sqrt{v}} = \lambda \qquad \mu - r = \lambda(\sigma\sqrt{v})$$

temperature

Expected return is high when volatility or trading frequency is large.

- Kyle Obizhaeva

1976: Calibration
The Invention of Implied Volatility

- Latane and Rendelman suggest fitting option market prices to the Black-Scholes formula and extracting the WISD (weighted implied standard deviation of the stock). They then suggest calculating hedge ratios from the model using the WISD.

- "… the WISD is generally a better predictor of future variability than standard deviation predictors based on historical data."

- But implied volatilities are unstable.
  This process must be repeated as the implied volatility keeps changing, so there is *something not quite right*.

- Implied volatilities tell you that if you believe the model, given an option price, this is what the future must be like. But it isn't.

- Nevertheless, from now on everyone calibrates (and recalibrates) models.

- Most people don't even realize it was an invention.

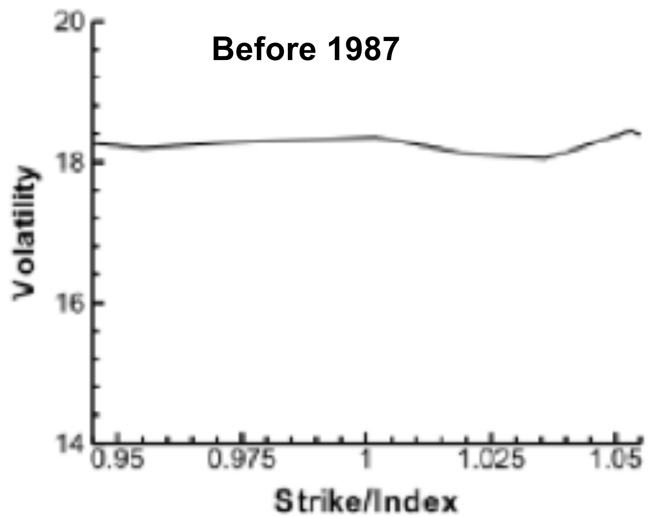
Trading Volatility is Now a Possibility

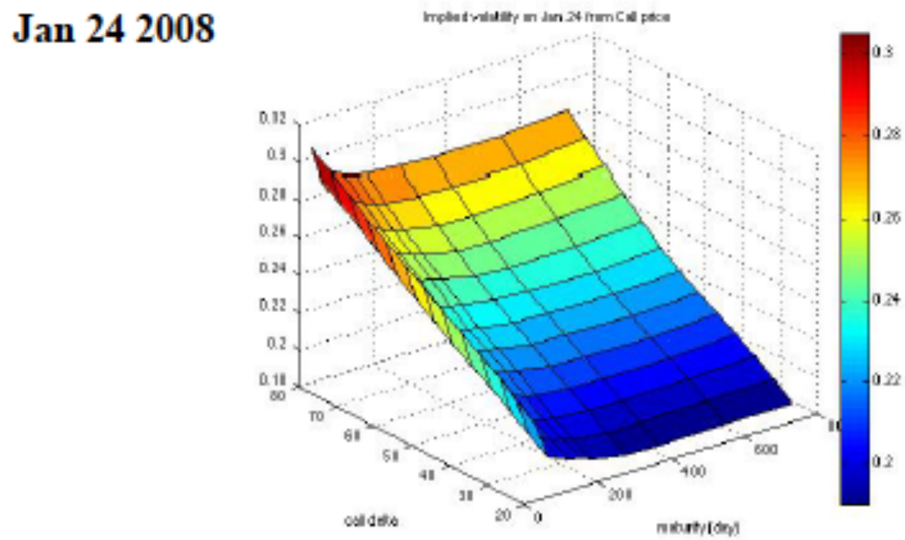
- BSM says that
  - Implied volatility is an estimate of future volatility.
  - But it keeps changing.
  - So you can't **really** replicate an option
  - Instead, you can speculate on volatility, using options.
- If you use the model, options now become a way of trading volatility rather than speculating on the stock price.
- Volatility as an asset class.

1977: Vašíček

- Modeling the yield curve – extension of Black-Scholes to parameters rather than assets.
- You can value options on two stock independently, but you cannot value options on two Treasury bonds independently.
- There are no-arbitrage constraints on bond prices.
- And so on … to other extensions of the hedging paradigm for 40 years …

1987: The Smile

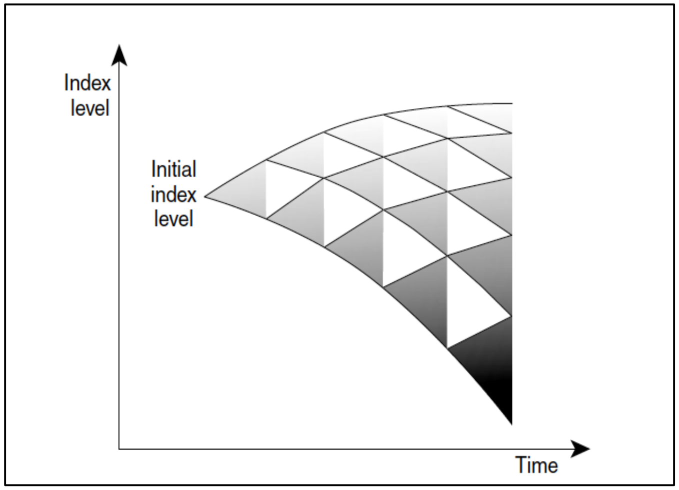


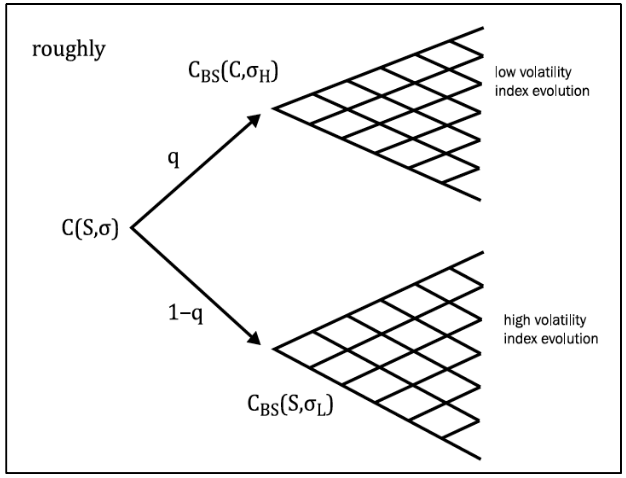


- When you fit the model to different option strikes, each one implies a different future volatility for the underlying stock. But in the model a stock can only have one volatility. *Now something is really wrong* – the BS model can NOT accommodate different volatilities for the same stock.

- Nevertheless, people keep using the model inconsistently to estimate hedge ratios as they calibrate the model to a given option price.

1994 - present: The Smile





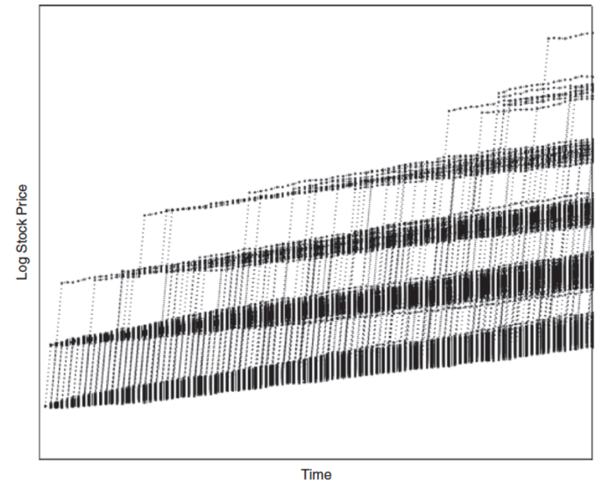


FIGURE 24.3 A Monte Carlo Simulation of the Log Stock Prices in the Jump-Diffusion Model

- Extensions: local volatility, stochastic volatility, jumps …
- More complexity without accurate knowledge of the parameters.
- Use market prices to imply the parameters - e.g. the volatility of volatility in a calibrated stochastic volatility model.
- But markets change and these implied parameters are themselves unstable and random, but there are now more of them. So, for example, vol of vol is now stochastic.
- So now *using the model* you have a market for trading volatility of volatility

The Future

- An infinite regress of models?
- Each model is inadequate, introduces new parameters, which, as the model is embraced, become quantities the market can speculate and trade on.
- "Derivatives are not (really) derivative, except at expiration."
- It takes all of these securities —
  - stocks, options, options on options, volatility, volatility of volatility …
- to define the possibilities of the market.
- These instruments are not truly derivative. The probabilistic approach to distributions is a fallacy. The "probabilities" only come into existence after an event, which is the offering a price. (Ayache, *The Medium of Contingency*)
- "All we have in the market are prices of contingent claims of varying complexity. Probabilistic models and pricing kernels and derivative valuation tools are only internal episodes that we require in order locally and always imperfectly to hedge something. We have to keep in mind that those episodes are present and useful only insofar as they will be recalibrated."

Human Affairs