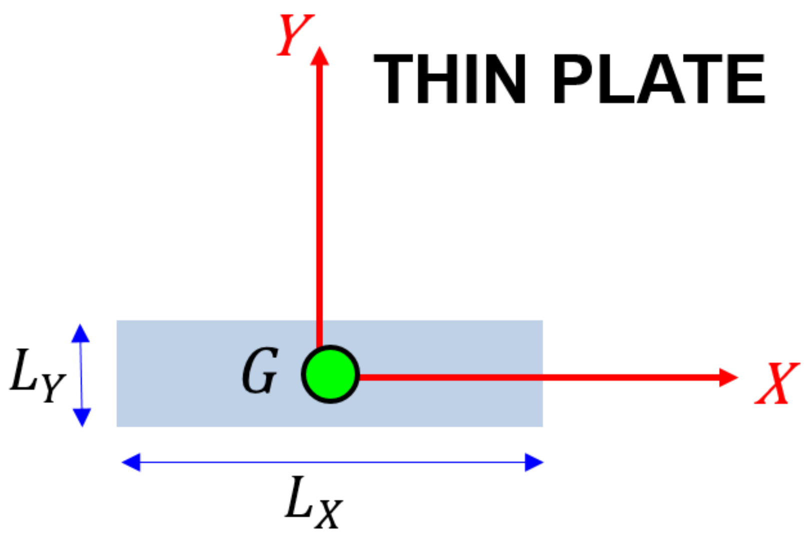
Inertia properties of a 2 blade propeller

What we're going to do:

In this FAQ, we're going to explore the inertia properties of a 2 bladed propeller. We'll approximate the propeller by a rectangular plate



WHY are we doing this?

- explore how the inertia matrix (relative to the XY-frame) changes as the propeller spins

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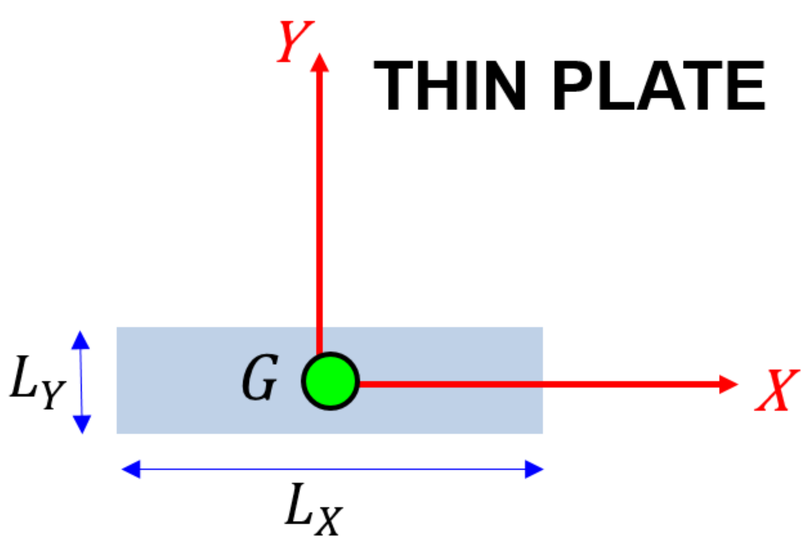
Consider a thin Rectangular plate:



In this figure we have a G-frame attached to the Centre of mass of the plate, Let's calculate the inertia of the plate about this G-frame

```
syms Lx Ly m
```

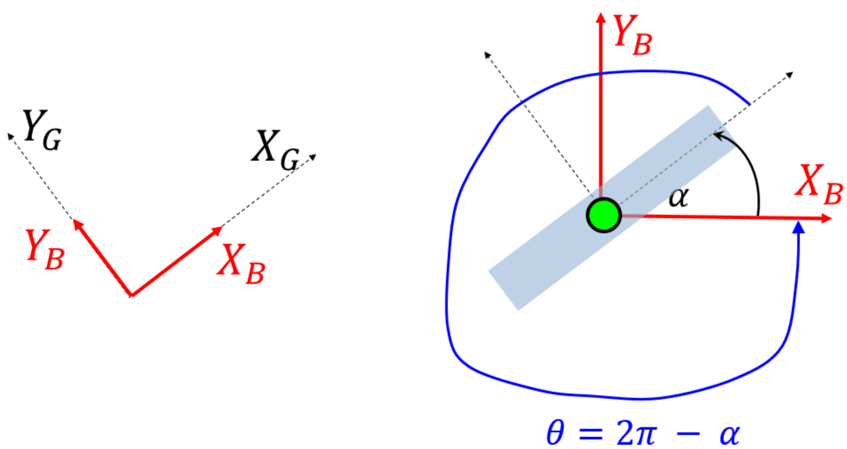
```
% create an instance of a thin rectangular plate class
TRP_OBJ = inertia_thin_rect_plate_CLS(Lx, Ly, m);

% look at the Inertia matrix for the G-frame
I_LOCAL = TRP_OBJ.get_I()
```

I_LOCAL =

$$\begin{pmatrix} \frac{\mathrm{Ly}^2 m}{12} & 0 & 0 \\ 0 & \frac{\mathrm{Lx}^2 m}{12} & 0 \\ 0 & 0 & \frac{m\,(\mathrm{Lx}^2+\mathrm{Ly}^2)}{12} \end{pmatrix}$$

Consider an arbitrarily orientated propeller:

Consider the following arbitrarily orientated propeller system:




```
% create a PASSIVE rotation object
syms alpha
BLUE_OBJ  = bh_rot_passive_G2B_CLS({'D1Z'}, [ (2*pi - alpha) ], 'SYM');

% extract the PASIVE rotation matrix bRg
BLUE_bRg = BLUE_OBJ.get_R1
```

BLUE_bRg =

$$\begin{pmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
% calculate the inertias relative to the XB,YB frame
gI = I_LOCAL;
```

```
bRg = BLUE_bRg;
Ib_blade  = bRg * gI * bRg.'
```

Ib_blade =

$$\begin{pmatrix} \frac{m\,\mathrm{Lx}^2\sin(\alpha)^2}{12}+\frac{m\,\mathrm{Ly}^2\cos(\alpha)^2}{12} & \sigma_1 & 0 \\ \sigma_1 & \frac{m\,\mathrm{Lx}^2\cos(\alpha)^2}{12}+\frac{m\,\mathrm{Ly}^2\sin(\alpha)^2}{12} & 0 \\ 0 & 0 & \frac{m\,(\mathrm{Lx}^2+\mathrm{Ly}^2)}{12} \end{pmatrix}$$

where

$$\sigma_1 = \frac{\mathrm{Ly}^2\,m\cos(\alpha)\sin(\alpha)}{12} - \frac{\mathrm{Lx}^2\,m\cos(\alpha)\sin(\alpha)}{12}$$

Let's plot how the these terms vary with alpha.

substitute numeric values for Lx and Ly and mass

```
Lx_num = 1;
Ly_num = 0.1;
m_num = 1;
I_LOCAL_num = subs(I_LOCAL, [Lx, Ly, m], [Lx_num, Ly_num, m_num])
```

I_LOCAL_num =

$$\begin{pmatrix} \frac{1}{1200} & 0 & 0 \\ 0 & \frac{1}{12} & 0 \\ 0 & 0 & \frac{101}{1200} \end{pmatrix}$$

```
Ib_blade_num = subs(Ib_blade, [Lx, Ly, m], [Lx_num, Ly_num, m_num])
```

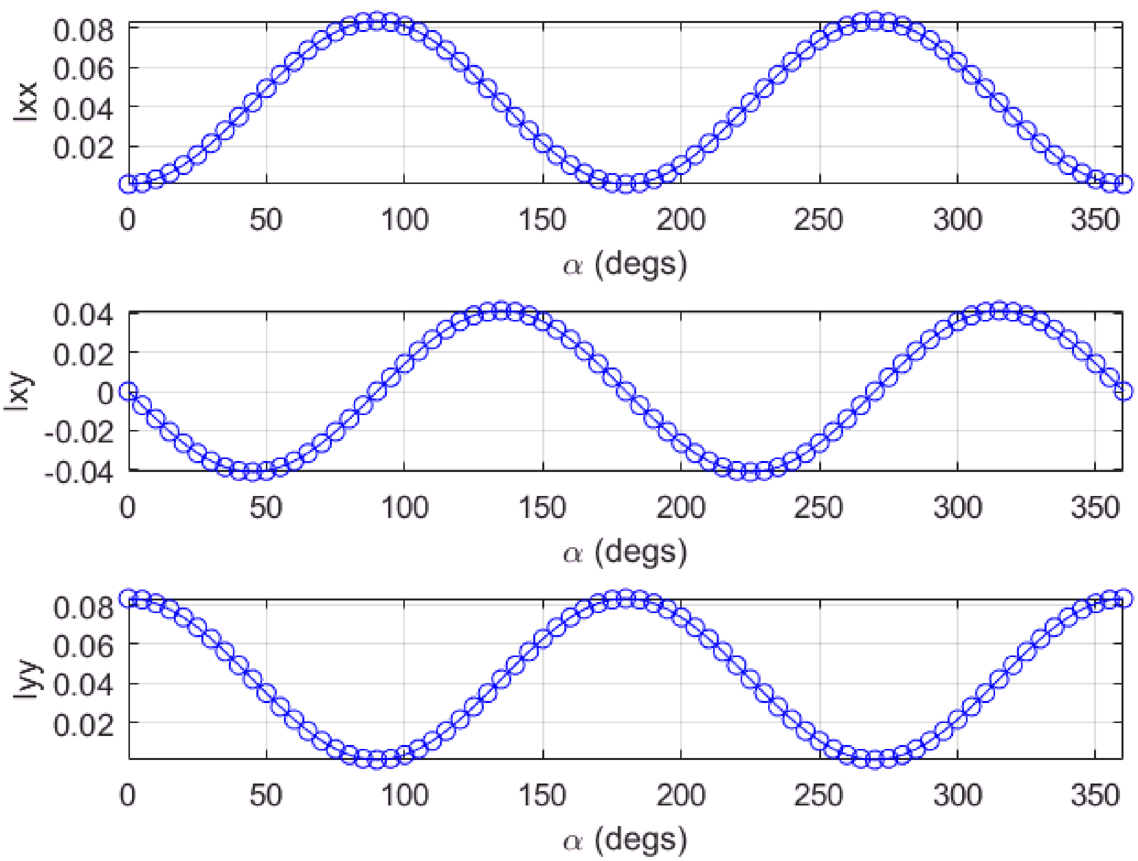
Ib_blade_num =

$$\begin{pmatrix} \frac{\cos(\alpha)^2}{1200}+\frac{\sin(\alpha)^2}{12} & -\frac{33\cos(\alpha)\sin(\alpha)}{400} & 0 \\ -\frac{33\cos(\alpha)\sin(\alpha)}{400} & \frac{\cos(\alpha)^2}{12}+\frac{\sin(\alpha)^2}{1200} & 0 \\ 0 & 0 & \frac{101}{1200} \end{pmatrix}$$

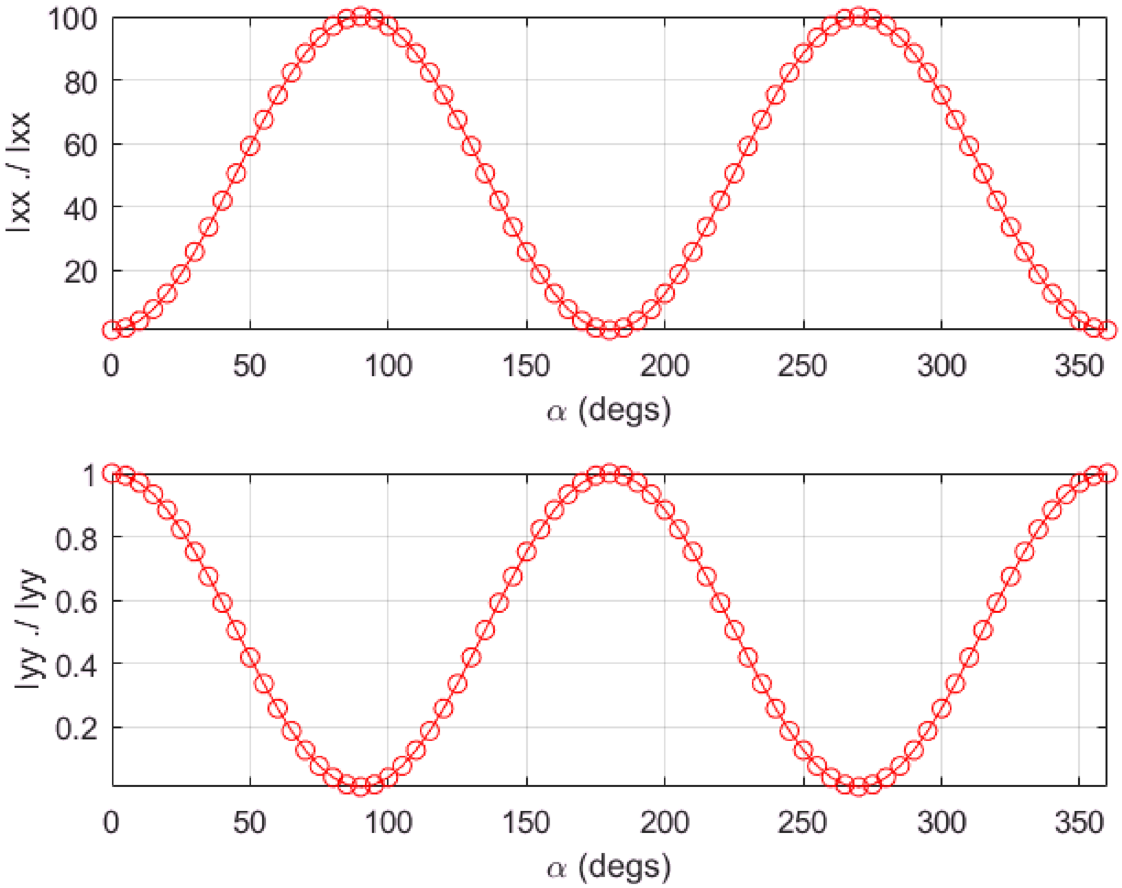
OK: now let's calculate the inertia matrix for different values of α

```
alpha_deg_num = [0:5:360];
alpha_rad_num = (pi/180)*alpha_deg_num;
for kk=1:length(alpha_rad_num)
```

```
    alpha_num = alpha_rad_num(kk);
    tmp_I     =  subs(Ib_blade_num, alpha, alpha_num);

    bh.Ixx(kk) = tmp_I(1,1);
    bh.Iyy(kk) = tmp_I(2,2);
    bh.Ixy(kk) = tmp_I(1,2);
end

% OK plot it
figure
subplot(3,1,1);
   plot(alpha_deg_num, bh.Ixx, 'ob-');
        axis('tight'); grid('on'); xlabel('\alpha (degs)'); ylabel('Ixx');
subplot(3,1,2);
   plot(alpha_deg_num, bh.Ixy, 'ob-');
        axis('tight'); grid('on'); xlabel('\alpha (degs)'); ylabel('Ixy');
subplot(3,1,3);
   plot(alpha_deg_num, bh.Iyy, 'ob-');
   axis('tight'); grid('on'); xlabel('\alpha (degs)'); ylabel('Iyy');
```




```
% OK plot the terms normalised by the local pose inertias
figure
subplot(2,1,1);
   plot(alpha_deg_num, bh.Ixx/I_LOCAL_num(1,1), 'or-');
        axis('tight'); grid('on'); xlabel('\alpha (degs)'); ylabel('Ixx ./ Ixx');
subplot(2,1,2);
   plot(alpha_deg_num, bh.Iyy/I_LOCAL_num(2,2), 'or-');
   axis('tight'); grid('on'); xlabel('\alpha (degs)'); ylabel('Iyy ./ Iyy');
```



100
80
60
40
20
Ixx ./ Ixx
0
50
100
150
200
250
300
350
α (degs)
1
0.8
0.6
0.4
0.2
Iyy ./ Iyy
0
50
100
150
200
250
300
350
α (degs)